

# A Novel Method Based on Two Different Thicknesses of The Sample for Determining Complex Permittivity of Materials Using Electromagnetic Wave Propagation in Free Space at X-Band

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## Abstract

In this paper, we present a method for determining complex permittivity of materials using two different thicknesses of the sample placed in free space. The proposed method is based on the use of transmission having the same geometry with different thicknesses with the aim to determine the complex propagation constant ( $\gamma$ ). The reflection and transmission coefficients ( $S_{11}$  and  $S_{21}$ ) of material samples are determined using a free-space measurement system. The system consists of transmit and receive horn antennas operating at X-band. The complex permittivity of materials is calculated from the values of  $\gamma$ , in turns received from  $S_{11}$  and  $S_{21}$ . The proposed method is tested with different material samples in the frequency range of 8.0 – 12.0 GHz. The results show that the complex permittivity determination of low-loss material samples is more accurate than that of high-loss ones. However, the dielectric loss tangent of high-loss material samples is negligibly affected.

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## 1. Introduction

The complex propagation constant is determined from scattering S-parameters measurements performed on two lines (Line-Line Method) having the same characteristic impedance but different lengths [1]. Once the parameters are measured either the ABCD [2] or wave cascading matrix (WCM) [3-5] may be used for determining complex propagation constant. The proposed method for determining complex permittivity of materials are structure to connected with device

measurements such as printed circuit board (PCB) materials [6-12]. Although the proposed methods are simple, quick, and reliable to use. However, it has drawbacks such as the material samples to determine the complex permittivity require structures the type printed circuit board. The measurement of complex permittivity of material can be made by using the transmission/reflection method developed by Weir [13]. The method for determining S-parameters of material in free space are nondestructive and contactless; hence, they are especially suitable for measurement of the complex permittivity ( $\epsilon^*$ ) and complex permeability ( $\mu^*$ ) of material under

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high-temperature conditions. The most popular methods for determining the parameter of materials are proposed in [14-21]. The errors in free-space measurements are presumed to be due to diffraction effects at the edges of the sample and multiple reflections between the antennas. Diffraction effects at the edges of the sample are minimized by using spot-focusing horn lens antennas as transmitters and receivers. The method proposed by D. K. Ghodgaonkar et al. [14] have developed a free-space TRL (thru, reflect, line) calibration technique which eliminates errors due to multiple reflections. This method is especially suitable for quick, routine, and broad-band measurement of complex permittivity of high-loss materials. However, for materials with dielectric loss tangent less than 0.1, the loss factor measurements are found to be inaccurate because of errors in reflection and transmission coefficient measurements.

In this paper, we propose a method in free space for determining complex permittivity of materials based on the use of transmission having the same geometry with different

thicknesses. Diffraction effects at the edges of the sample and multiple reflections between the antennas are minimized by using two different thicknesses of the sample placed in free space. Our results indicate that the permittivity of material is quite stable in the frequency range of 8.0 – 12.0 GHz. In addition, for materials with dielectric loss tangent less than 0.1, the loss factor measurements are accuracy in the entire frequency band.

The next section describes the theory of our method in detail. The modeling and results are presented in section 3. Finally, section 4 concludes this paper.

### 2. Theory

The complex permittivity of materials is defined as

$$\epsilon^* = \epsilon' - j\epsilon'' = \epsilon'(1 - j\tan\delta_\epsilon) \tag{1}$$

where,  $\epsilon'$  and  $\epsilon''$  are the real and imaginary parts of complex permittivity, and  $\tan\delta_\epsilon$  is the dielectric loss tangent.

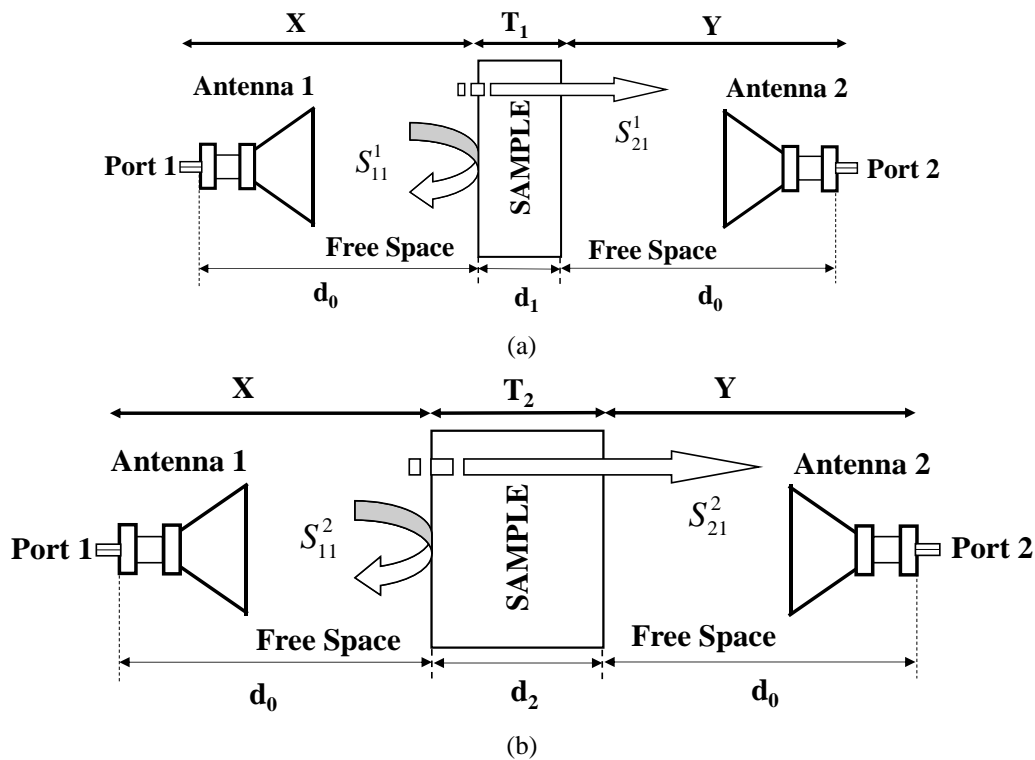


Figure 1. Schematic diagram of two transmissions (a) and (b).

Figure 1 shows two planar sample of thicknesses  $d_1$  and  $d_2$  ( $d_2 > d_1$ ) placed in free space. For both transmissions (a) and (b), the determined two port parameters expressed in ABCD matrix form can be considered as a product of three parts: an input matrix  $X$ , including the input coax-to-antenna transition, transmission  $T$ , and an output matrix  $Y$ , including the output coax-to-antenna transition. It can be shown that the  $M_1$  and  $M_2$  matrices are related to  $X$ ,  $T$  and  $Y$  by the following equations [2]:

$$M_1 = XT_1Y \quad (2)$$

$$M_2 = XT_2Y \quad (3)$$

where  $M_i$ ,  $X$ ,  $T_i$ , and  $Y$  are ABCD matrices for the corresponding sections as in the Figure 1.  $M_i$  can be related to measurable scattering parameters [22] by equation (4).

$$M_i = \frac{1}{s_{21}^i} \begin{pmatrix} s_{12}^i s_{21}^i - s_{11}^i s_{22}^i & s_{11}^i \\ -s_{22}^i & 1 \end{pmatrix} \quad (4)$$

The cascade matrix  $T_i$  of the homogenous transmission line  $i$ , is defined as

$$T_i = \begin{pmatrix} e^{-\gamma d_i} & 0 \\ 0 & e^{\gamma d_i} \end{pmatrix} \quad (5)$$

where  $\gamma$  and  $d_i$  are the complex propagation constant and length of the line. Multiplying the matrix  $M_1$  by the inverse matrix of  $M_2$ , we obtain (6)

$$M_1 M_2^{-1} = XT_1 T_2^{-1} X^{-1} \quad (6)$$

In (6), notice that  $M_1 M_2^{-1}$  is the similar transformation of  $T_1 T_2^{-1}$ . Using the fact that the trace, which is defined as the sum of the diagonal elements, does not change under the similar transformation in the matrix calculation, we can deduce (7)

$$\text{Tr}(M_1 M_2^{-1}) = \text{Tr}(T_1 T_2^{-1}) = 2 \cosh(\gamma \Delta d) \quad (7)$$

where  $\Delta d = (d_2 - d_1)$  is the length difference of two transmission lines. The complex propagation constant is found from (8)

$$\gamma = \frac{\cosh^{-1} \left\{ \frac{1}{2} \text{Tr}(M_1 M_2^{-1}) \right\}}{\Delta d} \quad (8)$$

The real part of  $\gamma$  is unique and single valued, but the imaginary part of  $\gamma$  has multiple values. It is defined as

$$\gamma = \alpha + j\beta = \alpha + j \frac{(\Delta\varphi - 360n)}{\Delta d} \quad (9)$$

where  $\alpha$  and  $\beta$  are the real and imaginary parts of the complex propagation constant,  $n$  is an integer ( $n = 0, \pm 1, \pm 2, \dots$ ),  $\Delta\varphi$  is the reading of the instrument ( $-180^\circ \leq \Delta\varphi \leq 180^\circ$ ). The phase constant  $\beta$  is defined as

$$\beta = \frac{360}{\lambda_0} \sqrt{\epsilon'} \quad (10)$$

where  $\lambda_0$  is the wavelength in free space.

The phase shift of complex propagation constant is the difference between the phase angle  $\Delta\Phi$  measured with two material sample between the two antennas, namely:

$$\Delta\Phi = \Phi_2 - \Phi_1 \quad (11)$$

where  $\Phi_i = \frac{-360d_i \sqrt{\epsilon'}}{\lambda_0}$  is the phase angle of material sample ( $i = 1, 2$ ). Consequently the phase shift is given by

$$\Delta\Phi = \frac{-360\Delta d \sqrt{\epsilon'}}{\lambda_0} \quad (12)$$

On the other hand, it can be expressed from (9) and (10) as

$$\Delta\Phi = \Delta\varphi - 360n \quad (13)$$

Measurements at two frequencies can also be used to solve the phase ambiguity problem [23]. The frequencies are selected in a region such that the difference between dielectric constants,  $\epsilon_1$  at  $f_1$ ,  $\epsilon_2$  at  $f_2$ , is small enough to permit the following assumption, using (12) and (13):

$$\lambda_{01}(\Delta\varphi_1 - 360n_1) = \lambda_{02}(\Delta\varphi_2 - 360n_2) \quad (14)$$

where  $\lambda_{01}$  and  $\lambda_{02}$  are the wavelengths in free space at  $f_1$  and  $f_2$ , respectively, with  $f_1 < f_2$ ,  $n_1$  and  $n_2$  are the integers to be determined. For this purpose, a second equation is needed. This equation can be

$$n_2 - n_1 = k \tag{15}$$

where  $k$  is an integer.

The integers  $n_1$  and  $n_2$  can be either equal ( $k = 0$ ) or different ( $k = 1, 2, \dots$ ) depending on the frequency difference and dielectric properties and thickness of material under test. Therefore, two cases can be distinguished:

+  $k = 0$

$$n_1 = n_2 = \frac{\lambda_{01}\Delta\varphi_1 - \lambda_{02}\Delta\varphi_2}{360(\lambda_{01} - \lambda_{02})} \tag{16}$$

+  $k \neq 0$

$$n_1 = \frac{\lambda_{01}\Delta\varphi_1 - \lambda_{02}\Delta\varphi_2}{360(\lambda_{01} - \lambda_{02})} + k \frac{\lambda_{02}}{\lambda_{01} - \lambda_{02}} \tag{17}$$

with

$$n_2 = n_1 + k \tag{18}$$

The complex permittivity of the material is calculated from (7), we obtain

$$\varepsilon^* = \left( \frac{c\gamma}{j2\pi f} \right)^2 \tag{19}$$

where  $f$  is the frequency and  $c$  is the light velocity.

### 3. Modeling and results

#### 3.1. Modeling

In this part, using the Computer Simulation Technology (CST) software to model system which presented in section 2, matrix  $S$  are determined from this modeling.

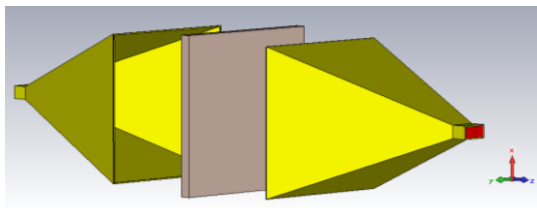


Figure 2. Modeling determining the parameters of material sample by CST.

In figure 2, two same pyramidal antennas are designed to operate well in the frequency range of 8.0 – 12.0 GHz. The gain and voltage standing wave ratio of the pyramidal horn antennas are 20 dBi and 1.15 at center frequency. In this model, the distance between the antenna and the material sample is 250mm ( $d_0 = 250\text{mm}$ ).

The two selected material samples have parameters as follows: The width and length of 150mm, the thicknesses of 7mm and 12mm. The complex permittivity of material samples:  $\varepsilon^* = 2.8 - j0$ ,  $\varepsilon^* = 2.8 - j0.14$ ,  $\varepsilon^* = 2.8 - j0.28$  and  $\varepsilon^* = 2.8 - j0.84$ . With  $\Delta d = 5\text{mm}$  is the length difference of two material samples. The frequencies  $f_1$  and  $f_2$  ( $f_1 < f_2$ ) are selected in the frequency range of 8.0 – 12.0 GHz. The results show that in the entire frequency band.

#### 3.2. Results

The reflection and transmission coefficients of two planar material samples are determined using the proposed model in section 3.1. The complex permittivity of material samples is calculated by equation (19) in section 2.

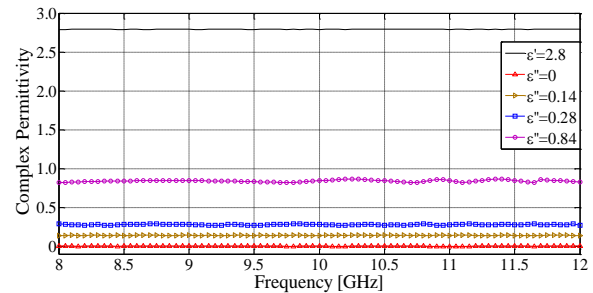


Figure 3. Complex permittivity of material samples ( $\Delta d = 5\text{mm}$ ).

Figure 3 shows the data obtained using the proposed method. The real part of the complex permittivity are quite stable and the mean error difference of 0.2% in the entire frequency band. The imaginary part of the complex permittivity are also stable and small the errors. The error of complex permittivity for materials with different dielectric loss tangent as shown in figure 4.

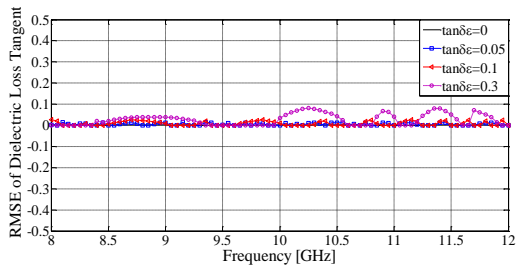


Figure 4. The root mean squared error of dielectric loss tangent the materials ( $\Delta d = 5\text{mm}$ ).

Figure 4 shows for materials with the dielectric loss tangent less than or equal to 0.1. The root mean squared error (RMSE) changes from 0 to 0.03. When dielectric loss tangent more than 0.1, the RMSE changes from 0 to 0.08. So, the results show that for materials with different dielectric loss tangent, the complex permittivity is nearly identical with the theoretical values. However, the dielectric loss tangent more than 0.1, the complex permittivity is effected by multiple reflections between the antennas. These errors are small and acceptable for high-loss materials.

The results show that the complex permittivity of low-loss material samples obtained by our method is more accurate than that calculated by the method proposed in [14]. However, with high-loss material samples, the root mean squared error of our method is larger than that of the method in [14].

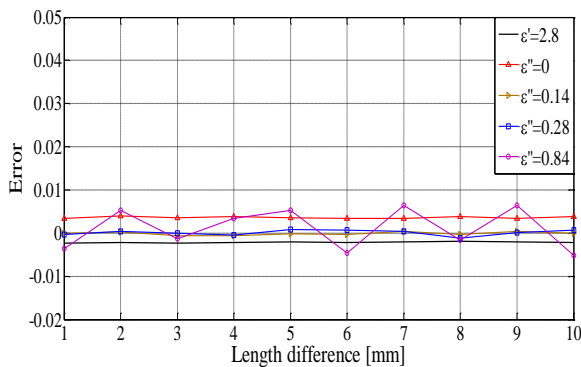


Figure 5. Error versus length difference of two transmission lines.

Figure 5 shows that the error versus the length differences of two transmission lines is very small, so that the complex permittivity of material samples is negligibly affected by the different thicknesses of those samples.

#### 4. Conclusion

We propose a method for determining the complex permittivity of materials using two different thicknesses of the sample in free space. The method consists of two antennas placed in free space and the two different thicknesses material samples placed in the middle of the two antennas. The results show that the permittivity of material is quite stable in the frequency range 8.0 - 12.0 GHz. In addition, the dielectric loss tangent of low-loss material samples is determined accurately by using proposed method. Our proposed method is especially suitable for determining complex permittivity of low-loss materials.

This method is applicable in many scientific fields such as: electronics, communications, metrology, mining, surveying, etc. Because this method is nondestructive and contactless, it can be used for broad-band measurement of permittivity under high-temperature conditions.

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