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Original Article

Combining Power Allocation and Superposition Coding for an Underlay Two-way Decode-and-forward Scheme

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Abstract: In this paper, we analyze an underlay two-way decode-and-forward scheme in which secondary relays use successive interference cancellation (SIC) technology to decode data of two secondary sources sequentially, and then generate a coded signal by superposition coding (SC) technology, denoted as SIC-SC protocol. The SIC-SC protocol is designed to operate in two time slots under effects from an interference constraint of a primary receiver and residual interference of imperfect SIC processes. Transmit powers provided to carry the data are allocated dynamically according to channel powers of interference and transmission, and a secondary relay is selected from considering strongest channel gain subject to increase in decoding capacity of the first data and decrease in collection time of channel state information. Closed-form outage probability expressions are derived from mathematical manipulations and verified by performing Monte Carlo simulations. An identical scheme of underlay two-way decodeand-forward relaying with random relay selection and fixed power allocations is considered to compare with the proposed SIC-SC protocol, denoted as RRS protocol. Simulation and analysis results show that the non-identical outage performances of the secondary sources in the proposed SIC-SC protocol are improved by increasing the number of the secondary relays and the interference constraint as well as decreasing the residual interference powers. Secondly, the performance of the nearer secondary source is worse than that of the farther secondary source. In addition, the proposed SIC-SC protocol outperforms the RRS comparison protocol, and effect of power allocations through channel powers is discovered. Finally, derived theory values are precise to simulation results.

Keywords: Successive interference cancellation, superposition coding, power allocation, underlay cognitive radio, non-orthogonal multiple access, outage probability.

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1. Introduction

Two-way communication is a protocol for exchanging data back and forth between users in which the users are both receiver and transmitter points [1, 2]. Energy and spectrum utilization efficiencies have been enhanced by two-way cooperation [1, 2]. Intermediate relays operating as amplify-and-forward (AF) and decode-and-forward (DF) devices process received signals before forwarding back to the users. With lower transmit powers, the noise-added data amplified is always sent to the desired users in the AF operation whereas the cooperative relays in the DF operation drop those data due to unsuccessful decoding. However, with higher transmit powers, the noises are cleared by the DF relays and then the desired users get a high success rate of decoding. Transmit frequency spectrum of the users and relays can be licensed or are shared by primary users (PUs) [3-5]. Spectrum sharing solution known as cognitive radio is to enhance bandwidth demand for mobile multimedia services and the explosive development of next-generation wireless networks such as wireless sensor networks, Internet-of-things (IoT), next-generation mobile networks ...where secondary users (SUs) collaborate with the PUs [3-5]. In the cognitive radio networks, the SUs can transmit at any time so that the interference powers at the PUs are limited under tolerable thresholds, denoted as underlay operation protocol [4, 5]. The tolerable thresholds are gifts of the primary networks sent to the secondary networks based on the quality-of-service (QoS) contracts. As a result, the transmit power of the SUs must be adjusted continuously as a function of tolerable interference thresholds and channel gains.

Investigations on performance of underlay two-way relaying systems have been considered in [7-11]. System performance of secondary two-way networks under an interference constraint of a primary receiver was improved by combining digital network coding and opportunity relay selection in which a selected secondary relay created a new data by XOR operation after decoding received data successfully [7]. Effects of multiple primary receivers have been investigated in [8, 9]. The authors in [10] took imperfect channel state information (CSI) from the SUs to the Pus into probability analyses. S. Solanki et. al in [11] exploited direct links to build adaptive protocols under average interference constraints. The transmit powers of the secondary sources and the secondary relays were set independently to maximum of achievable thresholds [7, 10, 11] or minimum of internal powers and mutual interference constraints [8, 9]. In addition, most of these investigations have been proposed on three-phase solutions, and therefore the bandwidth utilization is divided by three times.

The authors in [12] employed superposition coding (SC) at each source group and successive interference cancellation (SIC) at a relay to send lots of broadcast data whereas only using two time slots (two phases) in two-way relaying networks. The SC and SIC are core technologies in nonorthogonal multiple access (NOMA) systems [13]. The SC technology is used at the transmit source to merge signals with different powers based on distances to the destinations. The farther destinations are allocated by the higher signal powers. The nearby destinations apply the SIC technology to decode the derided signals by canceling the higher-powered signals and treating the lower-powered signals as interference [14]. However, decoding operations by the SIC technology can be imperfect because of residual interference signals [12].

Combination of the SC technology and the power allocation in the NOMA networks has been investigated in [12, 14-19]. Transmit powers under control of interference constraint [15, 18, 19] and maximum power limitation [12, 14-17] were allocated to fixed values to share a part of the total power to different users in multiple access operations. In [18], transmit power of a base station in multicast networks was achieved to maximum constraint following a min-max rule of overall maximum permissible interference powers and maximum transmission power whereas power allocation coefficient were constant values based on channel gains from the base station to multi users. A similar set-up model as [18] has also been considered in a recent study [19] to against eavesdroppers. Power allocation coefficients in the SC technology without constant values should be considered in multiple-access networks.

In this paper, we analyze an underlay two-way DF scheme with two secondary sources, multiple secondary relays and a primary receiver. In this scheme, the secondary relays use the SIC technology to decode the data of two secondary sources sequentially, and then generate a coded signal by the SC technology, denoted as SIC-SC protocol. By applying the uplink NOMA protocol, the SIC technology and the SC technology, the SICSC protocol operates in two time slots, and also suffers an interference constraint of the primary receiver and residual interference of the imperfect SIC processes. Transmit powers provided to carry the data are allocated dynamically according to channel powers of interference and transmission. A secondary relay is selected from considering strongest channel gain subject to increase in decoding capacity of the first data and decrease in collection time of CSIs. System performance of the SIC-SC protocol is evaluated by closed-form outage probability expressions. These outage probability analyses are verified by performing Monte Carlo simulations. An identical scheme of underlay two-way DF relaying with random relay selection and the fixed power allocations are considered to compare the proposed SIC-SC protocol, denoted as RRS protocol. The simulation and analysis results show contributions as follows. Firstly, with the SIC and SC technologies combining the power allocations, the non-identical outage performances of the secondary sources are improved when the number of the secondary relays and the interference constraint are increased as well as the residual interference powers are controlled to decrease. Secondly, the performance of the nearer secondary source is worse than that of the farther secondary source. Thirdly, the proposed SIC-SC protocol outperforms the RRS comparison protocol in terms of the outage probabilities, and discussions on effect of power allocations through channel powers are presented. Finally, derived theory values are precise to simulation results.

This paper is organized as follows. Section 2 presents a system model of an underlay two-way DF scheme. Section 3 analyzes outage probabilities of the proposed SIC-SC protocol and the RRS comparison protocol. Analysis and simulation results are presented in Section 4. Finally, Section 5 summarizes contributions.

2. System Model

Figure 1 presents a system model of an underlay two-way DF scheme in which two secondary sources SS_1 and SS_2 send corresponding data s_1 and s_2 to each other with the help of a closed group of N intermediate secondary relays SR_i, where $i \in \{1, 2, ..., N\}$. The secondary network nodes SS_1, SS_2 and SR_i have identical variances of the zero-mean white Gaussian noises (AWGN) (denoted as N_0), and are in an interference constraint of a primary receiver PR (denoted as I). The secondary relays SR_i are nearer to the secondary source SS_1 than the SS_2, and thus the secondary relays

SR_i use the SIC technology to decode the data s_1 firstly. In Figure 1, a direct transmission between secondary sources SS_1 and SS_2 is skipped by far distance or deep shadow fading [2, 20], and secondary and primary nodes are installed with a single antenna.

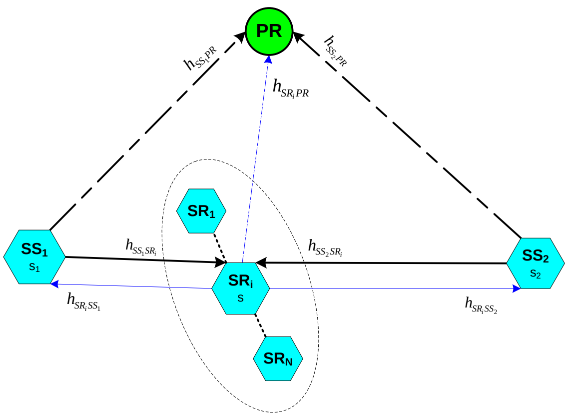

Figure 1. System model.

In Figure 1, h_{XY} denotes wireless channels of links X − Y which are modeled as complex normal distributions $h_{XY} \sim CN(0, \Psi_{XY})$ with zero means and normalized variances Ψ_{XY} (Ψ_{XY} are also the normalized powers of the channels) [21], where $X,Y \in \{SS_1, SS_2, SR_i, PR\}$. For the reason that the secondary relays SR_i are located in the closed cluster and are closer to SS_1 than SS_2, thus the normalized variances are set as $\Psi_{SS_1SR_i} = \Psi_{SR_iSS_1} = \Psi_1$, $\Psi_{SS_2SR_i} = \Psi_{SR_iSS_2} = \Psi_2$, $\Psi_{SR_iPR} = \Psi_3$, $\Psi_{SS_1PR} = \Psi_4$, $\Psi_{SS_2PR} = \Psi_5$ and $\Psi_1 > \Psi_2$ [22, 23]. As a result, $g_{XY} = |h_{XY}|^2$ are exponentially distributed random variables (RVs) with the probability density function (PDF) as $f_{g_{XY}}(x) = e^{-x/\Psi_{XY}} / \Psi_{XY}$ and the cumulative distribution function (CDF) as $F_{g_{XY}}(x) = 1 - e^{-x/\Psi_{XY}}$ [24] (see the equation (6-68)).

Traditionally, a set-up phase is performed according to the cooperative medium access control protocol [25]. The secondary nodes SS_1, SS_2 and SR_i can know perfectly the CSIs h_{SS_1PR}, h_{SS_2PR} and h_{SR_iPR} respectively by directly feedback channels from the primary receiver PR [22, 26] or by indirectly feedback channels from a third party [22, 27], $i \in \{1, 2, ..., N\}$. Firstly, the secondary sources SS_1 and SS_2 broadcast request-to-send messages (RTS) sequentially to the secondary relays SR_i with small transmit powers and low rates. The RTS messages contain its information such as the links to the PR. From receiving these RTS messages, the SR_i can estimate the fading channels $h_{SS_1SR_i}$ and $h_{SS_2SR_i}$, and then broadcast a helper-ready-to-send (HTS) message, which contains the $h_{SS_1SR_i}$ and h_{SR_iPR}, to the SS_1 and SS_2. Next, the SS_1 and SS_2 can estimate the fading channels $h_{SR_iSS_1}$ and $h_{SR_iSS_2}$ from the received HTS messages, respectively. Therefore, the SS_1 can know perfectly all fading channels to the secondary relays and the information link of the SS_2 to allocate the transmit powers and select the cooperative secondary relay. In addition, the selected relay can use the detected and estimated information to cancel interference. Finally, the SS_1 will send a clear-to-send message (CTS) including initial parameters to begin a next data transmission phase.

In the underlay cognitive radio schemes, the interference at the primary receiver PR from the secondary network are less than or equal the constraint I [28, 29]. Inequalities related to transmit powers and channel gains are obtained as

$$\begin{cases} P_{SS_1} g_{SS_1PR} + P_{SS_2} g_{SS_2PR} \le I \\ P_{SR_i} g_{SR_iPR} \le I \end{cases}, i \in \{1, 2, ..., N\}, \quad (1)$$

where P_{SS_1}, P_{SS_2} and P_{SR_i} are transmit powers of the SS_1, the SS_2 and the SR_i, respectively.

To maximize system performance of the secondary network, and coordinate balance manner between the transmit powers and channel gains from secondary sources SS_1 and SS_2 to the PR, we allocate the transmit powers

P_{SS_1}, P_{SS_2} and P_{SR_i} as $P_{SS_1} = I\Psi_5 \big/ \left(g_{SS_1PR} \left(\Psi_4 + \Psi_5 \right) \right)$, $P_{SS_2} = I\Psi_4 \big/ \left(g_{SS_2PR} \left(\Psi_4 + \Psi_5 \right) \right)$ and $P_{SR_i} = I \big/ g_{SR_iPR}$. It is worth noting that this allocation proposal is used in many published literature [7, 23, 30-33].

The operation principle of the SIC-SC protocol occurs in two time slots and is presented by mathematical models as follows. In the first time slot, the secondary sources SS_1 and SS_2 send the data s_1 and s_2, respectively, with the same carrier frequency to the secondary relays SR_i at the same time, where $i \in \{1, 2, ..., N\}$. The received signal at the secondary relays SR_i is stated as

$$y_{SR_i} = \sqrt{P_{SS_1}} s_1 h_{SS_1SR_i} + \sqrt{P_{SS_2}} s_2 h_{SS_2SR_i} + n_{SR_i}, \tag{2}$$

where $E\left\{ |s_1|^2 \right\} = E\left\{ |s_2|^2 \right\} = 1$ and n_{SR_i} denote the AWGNs at the SR_i with the same normalized variance N_0 ($E\{\cdot\}$ denote the expectation operator).

Since $\Psi_1 > \Psi_2$ (SR_i is closer to the SS_1 than the SS_2), based on the SIC technology, the SR_i will decode the data s_1 in (2) firstly by considering the signal $\sqrt{P_{SS_2}} s_2 h_{SS_2SR_i}$ as interference. From (2), the received signal-to-interference-plus-noise ratios (SINRs) at SR_i to decode s_1 is expressed as

$$\begin{aligned} \gamma_{SR_i \to s_1} &= \frac{P_{SS_1} g_{SS_1SR_i}}{P_{SS_2} g_{SS_2SR_i} + N_0} \\ &= \frac{\Psi_5 Q g_{SS_1SR_i} g_{SS_2PR}}{\Psi_4 Q g_{SS_2SR_i} g_{SS_1PR} + \left(\Psi_4 + \Psi_5 \right) g_{SS_1PR} g_{SS_2PR}}, \end{aligned} \tag{3}$$

where $Q = I/N_0$.

From acquisitions of perfect CSIs, the secondary source SS_1 only select one secondary relay (denoted as SR_n, $n \in \{1, 2, ..., N\}$) so that the decoding capacity is increased and the number of the pilot channels is decreased. The secondary relay SR_n is obtained as $\mathrm{SR}_n = \arg \max\limits_{i=1,2,...,N} g_{SS_1SR_i}$. As a result, the SINR at the SR_n in (3) can achieve higher value when comparing with relay selection randomly (take any secondary relay in a group of N secondary relays).

By the SIC procedure completely or partly, the interference part $\sqrt{P_{SS_1}} s_1 h_{SS_1SR_n}$ in (2) can be canceled after the data s_1 is decoded successfully, and thus, the received signal at the secondary relays SR_n after the SIC procedure is expressed as

$$\begin{aligned} y_{SR_n \to s_2} &= y_{SR_n} - \sqrt{P_{SS_1}} s_1 h_{SS_1SR_n} \\ &= \sqrt{P_{SS_2}} s_2 h_{SS_2SR_n} + \sqrt{\varepsilon I} h_n + n_{SR_n}, \end{aligned} \tag{4}$$

where $\sqrt{\varepsilon I}$ is a residual interference component at the SR_n due to imperfect SIC procedure; $\varepsilon = 0$ and $\varepsilon = 1$ express perfect and imperfect interference cancellation at the SR_n, respectively; h_n is modeled as an identical complex normal distribution $h_n \sim CN\left(0, \Psi_6\right)$ [12, 16] with zero mean and same normalized variance Ψ_6, and thus $g_n = |h_n|^2$ are also exponentially distributed RVs with the PDF as $f_{g_n}(x) = e^{-x/\Psi_6} \big/ \Psi_6$ and the CDF as $F_{g_n}(x) = 1 - e^{-x/\Psi_6}$ [24] (see the equations (6-68)).

The received SINR at the selected secondary relay SR_n to decode s_2 is obtained and manipulated as

$$\gamma_{SR_n \to s_2} = \frac{\Psi_4 Q g_{SS_2SR_n}}{g_{SS_2PR} \left(\varepsilon Q g_n + 1 \right) \left(\Psi_4 + \Psi_5 \right)}. \tag{5}$$

If the selected secondary relay SR_n decodes successfully both data s_1 and s_2, a coded data s is created by the SC technology as $s = s_1 \sqrt{\Psi_1 \big/ \left(\Psi_1 + \Psi_2 \right)} + s_2 \sqrt{\Psi_2 \big/ \left(\Psi_1 + \Psi_2 \right)}$. In this case, because the SR_n is nearer to the secondary source SS_1 than to the secondary source SS_2, thus, the SR_n sends the data s_2 to the SS_1 with the smaller power parameter $\Psi_2 \big/ \left(\Psi_1 + \Psi_2 \right)$ and vice versus.

In the second time slot, the SR_n broadcasts back the coded data s to two secondary sources

SS_1 and SS_2, and then the received signals at the secondary sources SS_k are obtained as

$$\begin{aligned} y_{\mathrm{SS}_k} &= \sqrt{P_{\mathrm{SR}_n}}\, s h_{\mathrm{SR}_n\mathrm{SS}_k} + n_{\mathrm{SS}_k} \\ &= \sqrt{P_{\mathrm{SR}_n}\, \Psi_1/(\Psi_1+\Psi_2)}\, s_1 h_{\mathrm{SR}_n\mathrm{SS}_k} \\ &\quad + \sqrt{P_{\mathrm{SR}_n}\, \Psi_2/(\Psi_1+\Psi_2)}\, s_2 h_{\mathrm{SR}_n\mathrm{SS}_k} + n_{\mathrm{SS}_k}, \end{aligned} \tag{6}$$

where n_{SS_k} denote AWGNs at the SS_k with the same normalized variance N_0, and $k \in \{1,2\}$.

We assume that the SS_k can perfectly cancel the known components including its data s_k, i.e., $\sqrt{P_{\mathrm{SR}_n}\, \Psi_k/(\Psi_1+\Psi_2)}\, s_k h_{\mathrm{SR}_n\mathrm{SS}_k}$ in (6). The received SINRs at the secondary sources SS_k are obtained to take s_l, where $l \in \{1,2\}$ and $l \neq k$, as

$$\gamma_{SS_k \to s_l} = \frac{P_{SR_n}\Psi_l \left|h_{SR_nSS_k}\right|^2}{N_0(\Psi_1+\Psi_2)} = \frac{Q g_{SR_nSS_k}\Psi_l}{g_{SR_nPR}(\Psi_1+\Psi_2)}. \tag{7}$$

Here, we received data rates at the SS_1, SS_2 and SR_n as

$$R_{Z\to z} = \frac{1}{2}\log_2(1+\gamma_{Z\to z})(\text{bits/s/Hz}), \tag{8}$$

where ½ shows that the proposed SIC-SC protocol operates in the two time slots, $Z \in \{SS_1, SS_2, SR_n\}$, $z \in \{s_1, s_2\}$ and $n \in \{1,2,...,N\}$.

For comparison purpose, we also consider a random relay selection (RRS) protocol with fixed power allocations where a collaborative SR_i is randomly selected in a group of N secondary relays for the two-way relaying, $i \in \{1,2,...,N\}$ [17, 34-36]. In particular, from (1), the transmit powers of the SS_1 and SS_2 are maximally set as $P_{SS_1}^{RRS} = \phi_1 I/g_{SS_1PR}$ and $P_{SS_2}^{RRS} = \phi_2 I/g_{SS_1PR}$ where $\phi_1+\phi_2=1$ [15]. In addition, the secondary relay SR_i applies the SC technology with power allocation parameters ϕ_3 and ϕ_4 to create a coded data as $s_{RRS} = s_2\sqrt{\phi_3} + s_1\sqrt{\phi_4}$, where $\phi_3+\phi_4=1$ and $\phi_3 \le \phi_4$ (the SS_1 is the nearer user to receive the data s_2) [14].

3. Outage Probability Analyses

Outage probability at a node Υ is defined as probability that the node Υ cannot decode successfully the desired data or the received data rate at the node Υ is less than a threshold data rate R_{th} [29,37] $\left(R_{\Upsilon\to z} \le R_{th},\ \Upsilon \in \{SS_1, SS_2, SR_n\}, z \in \{s_1, s_2\}, n \in \{1,2,...,N\}\right)$.

The selected secondary relay SR_n applies the SIC procedure which was assigned in the set up phase to decode successively the data from s_1 to s_2, the SS_1 cannot get the data s_2 in the three cases as: 1) the SR_n cannot decode the first-ordered data s_1 (denoted as $R_{SR_n\to s_1} < R_{th}$), 2) the SR_n gets the data s_1 successfully but does not decode the second-ordered data s_2 (denoted as $\left(R_{SR_n\to s_1} \ge R_{th}\right)\cap\left(R_{SR_n\to s_2} < R_{th}\right)$), or 3) the SR_n gets both s_1 and s_2 successfully but the SS_1 cannot decode the desired data s_2 in the second time slot (denoted as $\left(R_{SR_n\to s_1} \ge R_{th}\right)\cap\left(R_{SR_n\to s_2} \ge R_{th}\right)\cap\left(R_{SS_1\to s_2} < R_{th}\right)$. By summing the above cases, the outage probability of the secondary source SS_1 is expressed mathematically as

$$\begin{aligned} OP_{SS_1} &= \underbrace{\Pr\left[R_{SR_n\to s_1} < R_{th}\right]}_{\Delta_1} \\ &\quad + \underbrace{\Pr\left[\left(R_{SR_n\to s_1} \ge R_{th}\right)\cap\left(R_{SR_n\to s_2} < R_{th}\right)\right]}_{\Delta_2} \\ &\quad + \underbrace{\Pr\left[\begin{array}{l}\left(R_{SR_n\to s_1} \ge R_{th}\right)\cap\left(R_{SR_n\to s_2} \ge R_{th}\right) \\ \cap\left(R_{SS_1\to s_2} < R_{th}\right)\end{array}\right]}_{\Delta_3} \end{aligned} \tag{9}$$

where $\Pr[\Xi]$ mean probability operations of events Ξ.

Lemma 1: The probability Δ_1 is solved as

$$\Delta_1 = 1 + \frac{\Psi_5}{\Psi_4}\sum_{p=1}^{N}\binom{p}{N}\frac{(-1)^p}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)} \times\left(1 - \frac{\Psi_2\upsilon_4(p)}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)}\ln\left(\frac{\Psi_5\upsilon_3(p)}{\Psi_2\upsilon_4(p)}\right)\right), \quad (10)$$

where: $\upsilon_1 = \left(2^{2R_{th}} - 1\right)\Psi_4 / \Psi_5$; $\upsilon_2 = \left(2^{2R_{th}} - 1\right)\left(\Psi_4 + \Psi_5\right)/\left(\Psi_5 Q\right)$; functions $\upsilon_3(p)$ and $\upsilon_4(p)$ are defined respectively as $\upsilon_3(p) = \left(\Psi_1 + p\Psi_4\upsilon_2\right)/\left(\Psi_1\Psi_4\right)$ and $\upsilon_4(p) = p\upsilon_1/\Psi_1$; $\binom{p}{N}$ denotes the binomial coefficient $\left(\binom{p}{N} = \frac{N!}{p!(N-p)!}\right)$.

Proof: Proven in **Appendix A**.

Lemma 2: The probability Δ_2 is given in two cases of the perfect (ε = 0) and imperfect SIC (ε = 1) operations as formula at the top of next page.

In the formulas (11), $\Gamma(a,b)$ is an incomplete Gamma function [38] (see the equation (8.350.2)), $\upsilon_5 = \left(2^{2R_{th}} - 1\right)\left(\Psi_4 + \Psi_5\right)/\left(\Psi_4 Q\right)$, $\upsilon_6 = \left(\Psi_2 + \upsilon_5\Psi_5\right)/\left(\upsilon_5 Q\Psi_5\Psi_6\right)$ and a function $\upsilon_7(p) = \left(\upsilon_3(p) + \upsilon_5\upsilon_4(p)\right)/\left(\upsilon_5 Q\Psi_6\upsilon_4(p)\right)$.

Proof of the Lemma 2 is provided in **Appendix B**.

The event $\left(R_{SS_1\to s_2} < R_{th}\right)$ happens independently with the intersection event of $\left(R_{SR_n\to s_1} \ge R_{th}\right)$ and $\left(R_{SR_n\to s_2} \ge R_{th}\right)$, thus we have an equivalent representation of the probability Δ_3 in (9) as formulas (12).

$$\Delta_2 = \begin{cases} \dfrac{\Psi_5\upsilon_5}{\Psi_2 + \Psi_5\upsilon_5} - \dfrac{1}{\Psi_4}\displaystyle\sum_{p=0}^{N}\binom{p}{N}\frac{(-1)^p}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)} \times \left(\begin{array}{l} \dfrac{\upsilon_5\Psi_5^2}{\Psi_2 + \Psi_5\upsilon_5} \\ -\dfrac{\Psi_2\Psi_5\upsilon_4(p)}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)}\ln\left(\dfrac{(\Psi_2 + \upsilon_5\Psi_5)\upsilon_3(p)}{\upsilon_5\Psi_2\left(\upsilon_3(p) + \upsilon_4(p)\right)}\right)\end{array}\right), \varepsilon = 0 \\ 1 - \dfrac{e^{\upsilon_6}\Gamma[0,\upsilon_6]}{\Psi_5\Psi_6}\left\{\dfrac{\Psi_2}{\upsilon_6} - \dfrac{1}{\upsilon_5\Psi_4 Q}\displaystyle\sum_{p=0}^{N}\binom{p}{N}\frac{(-1)^p\left(\Psi_5\upsilon_3(p) - (\Psi_2 + \upsilon_5 Q\Psi_5\Psi_6)\upsilon_4(p)\right)}{\left(\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)\right)^2}\right\} - \dfrac{\Psi_5}{\Psi_4}\displaystyle\sum_{p=0}^{N}\binom{p}{N} \\ \times\dfrac{(-1)^p}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)} \times \left\{1 - \dfrac{\Psi_2\upsilon_4(p)}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)}\left(\ln\left(\dfrac{(\Psi_2 + \upsilon_5\Psi_5)\upsilon_3(p)}{\Psi_2\left(\upsilon_3(p) + \upsilon_5\upsilon_4(p)\right)}\right) - e^{\upsilon_7(p)}\Gamma\left[0,\upsilon_7(p)\right]\right)\right\}, \varepsilon = 1 \end{cases} \quad (11)$$

$$\begin{aligned}\Delta_3 &= \Pr\left[\left(R_{SR_n\to s_1} \ge R_{th}\right)\cap\left(R_{SR_n\to s_2} \ge R_{th}\right)\right] \\ &\quad\times\Pr\left[R_{SS_1\to s_2} < R_{th}\right] \\ &= \left(1 - \Delta_1 - \Delta_2\right)\times\Pr\left[R_{SS_1\to s_2} < R_{th}\right]\end{aligned} \quad (12)$$

The probability $\Pr\left[R_{SS_1\to s_2} < R_{th}\right]$ is solved with referring the formula (A.2) as

$$\begin{aligned}\Pr\left[R_{SS_1\to s_2} < R_{th}\right] &= \Pr\left[\frac{1}{2}\log_2\left(1 + \gamma_{SS_1\to s_2}\right) < R_{th}\right] \\ &= \Pr\left[\frac{Qg_{SR_nSS_1}\Psi_2}{g_{SR_nPR}\left(\Psi_1 + \Psi_2\right)} < 2^{2R_{th}} - 1\right] \\ &= \Pr\left[\frac{g_{SR_nSS_1}}{g_{SR_nPR}} < \frac{\left(\Psi_1 + \Psi_2\right)\left(2^{2R_{th}} - 1\right)}{\Psi_2 Q}\right] \\ &= \frac{\left(\Psi_1 + \Psi_2\right)\Psi_3\left(2^{2R_{th}} - 1\right)}{\Psi_1\Psi_2 Q + \left(\Psi_1 + \Psi_2\right)\Psi_3\left(2^{2R_{th}} - 1\right)}.\end{aligned} \quad (13)$$

Substituting (12) into (9) and using (13), the outage probability of the secondary source SS_1 is analyzed by a closed-form expression as

$$OP_{SS_1}=\frac{(\Psi_1+\Psi_2)\Psi_3\left(2^{2R_{th}}-1\right)+(\Delta_1+\Delta_2)\Psi_1\Psi_2 Q}{\Psi_1\Psi_2 Q+(\Psi_1+\Psi_2)\Psi_3\left(2^{2R_{th}}-1\right)} \quad (14)$$

where Δ_1 and Δ_2 are provided by the Lemmas 1 and 2, respectively.

The SS_2 cannot get the desired data s_1 in only the two cases as: 1) the SR_n cannot also decode the first-ordered data s_1, 2) the SR_n transmits the signals containing the decoded data s_1 but the SS_2 cannot get the s_1 (denoted as $\left(R_{SR_n\to s_1}\ge R_{th}\right)\cap\left(R_{SS_2\to s_1}<R_{th}\right)$. Therefore, the outage probability of the secondary source SS_2 is presented and solved as

$$\begin{aligned}OP_{SS_2}&=\Pr\left[R_{SR_n\to s_1}<R_{th}\right]\\&\quad+\Pr\left[\left(R_{SR_n\to s_1}\ge R_{th}\right)\cap\left(R_{SS_2\to s_1}<R_{th}\right)\right]\\&=\Delta_1+\Pr\left[R_{SR_n\to s_1}\ge R_{th}\right]\Pr\left[R_{SS_2\to s_1}<R_{th}\right]\\&=\Delta_1+(1-\Delta_1)\frac{(\Psi_1+\Psi_2)\Psi_3\left(2^{2R_{th}}-1\right)}{\Psi_1\Psi_2 Q+(\Psi_1+\Psi_2)\Psi_3\left(2^{2R_{th}}-1\right)}\\&=\frac{(\Psi_1+\Psi_2)\Psi_3\left(2^{2R_{th}}-1\right)+\Psi_1\Psi_2 Q\Delta_1}{\Psi_1\Psi_2 Q+(\Psi_1+\Psi_2)\Psi_3\left(2^{2R_{th}}-1\right)}\end{aligned} \quad (15)$$

where Δ_1 is provided by the Lemma 1.

From (13-14), we notice that by allocating the transmit power ratios for the data s_1 and s_2 in the SC-coded signal s, the decoding outage probabilities at the secondary sources SS_1 and SS_2 from the selected secondary relay SR_n are identical, i.e. $\Pr\left[R_{SS_1\to s_2}<R_{th}\right]=\Pr\left[R_{SS_2\to s_1}<R_{th}\right]$.

In addition, from (14-15), we have a result as $OP_{SS_1}\ge OP_{SS_2}$ due to additional data decoding of s_2 at the selected secondary relay in the OP_{SS_1}.

Finally, the outage probabilities of the SS_1 and SS_2 in the RRS protocol (denoted as $OP_{SS_1}^{RRS}$ and $OP_{SS_2}^{RRS}$) are expressed and solved in the similar approach as in the proposed SIC-SC protocol. Hence, these outage probabilities are obtained as

$$\begin{aligned}OP_{SS_1}^{RRS}&=\underbrace{\Pr\left[R_{SR_i\to s_1}<R_{th}\right]}_{\Delta_4}\\&\quad+\underbrace{\Pr\left[\left(R_{SR_i\to s_1}\ge R_{th}\right)\cap\left(R_{SR_i\to s_2}<R_{th}\right)\right]}_{\Delta_5}\\&\quad+\Pr\left[\begin{array}{l}\left(R_{SR_i\to s_1}\ge R_{th}\right)\cap\left(R_{SR_i\to s_2}\ge R_{th}\right)\\\cap\left(R_{SS_1\to s_2}<R_{th}\right)\end{array}\right]\\&=\frac{\left(2^{2R_{th}}-1\right)\Psi_3+\phi_3\Psi_1 Q(\Delta_4+\Delta_5)}{\left(2^{2R_{th}}-1\right)\Psi_3+\phi_3\Psi_1 Q}\end{aligned} \quad (16)$$

$$\begin{aligned}OP_{SS_2}^{RRS}&=\Pr\left[R_{SR_i\to s_1}<R_{th}\right]\\&\quad+\Pr\left[\left(R_{SR_i\to s_1}\ge R_{th}\right)\cap\left(R_{SS_2\to s_1}<R_{th}\right)\right]\\&=\frac{\left(2^{2R_{th}}-1\right)\Psi_3+\phi_4\Psi_2 Q\Delta_4}{\left(2^{2R_{th}}-1\right)\Psi_3+\phi_4\Psi_2 Q}\end{aligned} \quad (17)$$

where Δ_4 and Δ_5 are inferred from Lemmas 1 and 2, and are presented by the below formula (18) and the formula (19) at the top of this page.

$$\begin{aligned}\Delta_4&=1-\frac{\Psi_5}{\Psi_4(\tau_3\Psi_5-\tau_4\Psi_2)}\\&\quad\times\left(1-\frac{\tau_4\Psi_2}{\tau_3\Psi_5-\tau_4\Psi_2}\ln\left(\frac{\tau_3\Psi_5}{\tau_4\Psi_2}\right)\right)\end{aligned} \quad (18)$$

In the formulas (18-19), $\tau_1=\left(2^{2R_{th}}-1\right)\phi_2/\phi_1$, $\tau_2=\tau_1/(\phi_2 Q)$, $\tau_3=(\Psi_1+\tau_2\Psi_4)/(\Psi_1\Psi_4)$, $\tau_4=\tau_1/\Psi_1$, $\tau_5=\tau_5\phi_1/\phi_2$, $\tau_6=(\Psi_2+\tau_5\Psi_5)/(\tau_5\Psi_5\Psi_6 Q)$ and $\tau_7=(\tau_3+\tau_4\tau_5)/(\tau_4\tau_5\Psi_6 Q)$.

$$\Delta_5 = \begin{cases} \dfrac{\Psi_5}{\Psi_4\left(\tau_3\Psi_5 - \tau_4\Psi_2\right)} \times \left(\dfrac{\tau_5\Psi_5}{\Psi_2 + \tau_5\Psi_5} - \dfrac{\tau_4\Psi_2}{\tau_3\Psi_5 - \tau_4\Psi_2} \ln\left(\dfrac{\tau_3\left(\Psi_2 + \tau_5\Psi_5\right)}{\left(\tau_3 + \tau_4\tau_5\right)\Psi_2} \right) \right) & ,\varepsilon = 0 \\ \dfrac{e^{\tau_6}\Gamma\left[0,\tau_6\right]}{\Psi_5\Psi_6} \left\{ \dfrac{1}{\tau_5\Psi_5 Q} - \dfrac{\Psi_2}{\tau_6} - \dfrac{\tau_3\Psi_5 - \tau_4\left(\Psi_2 + \tau_5\Psi_5\Psi_6 Q\right)}{\tau_5\Psi_4 Q\left(\tau_3\Psi_5 - \tau_4\Psi_2\right)^2} \right\} + \dfrac{\Psi_5}{\Psi_4\left(\tau_3\Psi_5 - \tau_4\Psi_2\right)} & \\ \qquad \times \left\{ 1 - \dfrac{\tau_4\Psi_2}{\tau_3\Psi_5 - \tau_4\Psi_2} \left(\ln\left(\dfrac{\tau_3\left(\Psi_2 + \tau_5\Psi_5\right)}{\left(\tau_3 + \tau_4\tau_5\right)\Psi_2} \right) - e^{\tau_7}\Gamma\left[0,\tau_7\right] \right) \right\} & ,\varepsilon = 1 \end{cases} \quad (19)$$

4. Results and Discussions

This section presents analysis and simulation results of the proposed SIC-SC protocol and the RRC comparison protocol with two cases of perfect SICs ($\varepsilon = 0$) and imperfect SICs ($\varepsilon = 1$). The Monte Carlo simulation results are made to verify the theoretical derivations in the section 3. The values for $\Psi_m, m \in \{1,2,3,4,5,6\}$ are referenced in [12, 18, 29, 39, 40]. In all the subsequent results, the threshold data rate R_{th} is fixed to 3 (bps/Hz) and the normalized variance of the additive white Gaussian noises (N_0) is set to 1. In addition, blue and red markers denote simulated values of the secondary sources SS_1 and SS_2, respectively, and black solid lines present theoretical analyses.

Figure 2 presents the outage probabilities of the secondary sources SS_1 and SS_2 in the protocols SIC-SC and RRS versus Q (dB) when $\Psi_1 = 20(\text{dB}), \Psi_2 = 10(\text{dB}), \Psi_3 = 1(\text{dB}), \Psi_4 = 1(\text{dB})$, $\Psi_5 = 0.5(\text{dB}), \Psi_6 = -5(\text{dB})$ and number of the secondary relays is examined at 5 and 8 $\left(N \in \{5,8\}\right)$. Power allocation parameters for the RRS comparison protocol are set to fixed values as $\phi_1 = 0.3, \phi_2 = 1 - \phi_1 = 0.7, \phi_3 = 0.4$ and $\phi_4 = 1 - \phi_3 = 0.6$. As shown in the Figure 2, some observations for the proposed SIC-SC protocol are listed as follows. Firstly, the outage probabilities of the secondary source SS_1 and SS_2 decrease when the number of the secondary relays N and interference constraint Q increase in two cases of the perfect SICs and imperfect SICs. Secondly, these probabilities move to saturation values at the high Q regions, e.g. $Q > 30$ (dB) when N = 8. Thirdly, the outage performance of the secondary source SS_2 is better than that of the secondary source SS_1. Fourthly, the outage probability of the secondary source SS_2 is not affected by the cases of the SIC operations and the secondary source SS_1 achieves smaller outage probabilities in the perfect SICs. Fifthly, the proposed SIC-SC protocol outperforms the RRS comparison protocol where the random relay selection and fixed power allocations are considered. Finally, the derived theory values (black solid lines) are precise to the simulation ones (marker symbols). These conclusions are explained by increasing the diversity amount and the transmit powers. In addition, the received SINRs at the secondary source SS_1 are weakened by combining effects of the interference component from the data-carried signal, the residual interference by imperfect SICs and interference constraints of the primary network whereas the perfect and imperfect SIC events existed in the received SINR $\gamma_{SR_n \to s_2}$ as in (5) and (8) are not considered in taking the own data s_2 of the secondary source SS_2 (see the formula (15)). One more thing, the RRS protocol does not depend on the number of the secondary relays and can be used if there is at least one cooperative secondary relay.

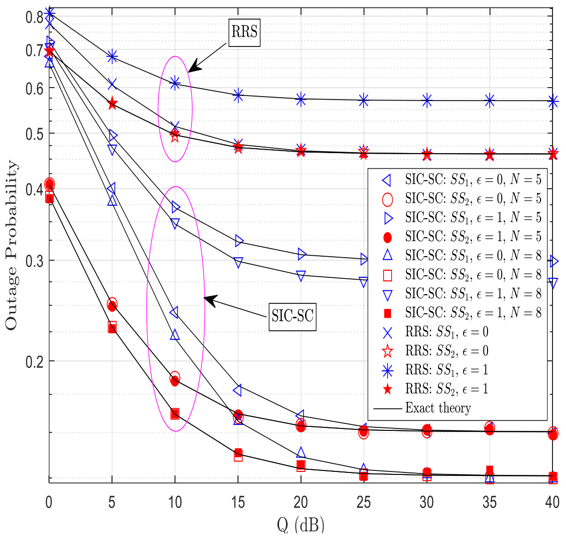

Figure 2. Outage probabilities of the SIC-SC and RRS protocols versus Q (dB) when $\Psi_1 = 20$ (dB), $\Psi_2 = 10$ (dB), $\Psi_3 = 1$ (dB), $\Psi_4 = 1$ (dB), $\Psi_5 = 0.5$ (dB), $\Psi_6 = -5$ (dB), $\phi_1 = 0.3$, $\phi_2 = 1 - \phi_1 = 0.7$, $\phi_3 = 0.4$, $\phi_4 = 1 - \phi_3 = 0.6$ and $N \in \{5, 8\}$.

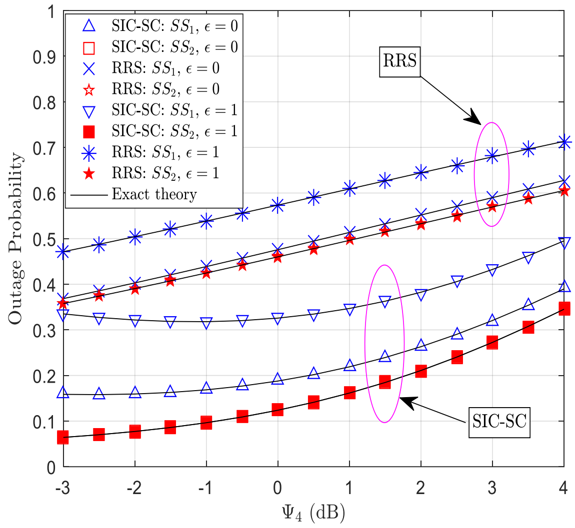

Figure 3. Outage probabilities of the SIC-SC and RRS protocols versus Ψ_4 (dB) when $\Psi_1 = 20$ (dB), $\Psi_2 = 10$ (dB), $\Psi_3 = 1$ (dB), $\Psi_5 = 0.5$ (dB), $\Psi_6 = -5$ (dB), $\phi_1 = 0.3, \phi_2 = 0.7, \phi_3 = 0.4, \phi_4 = 0.6$, $N = 8$ and $Q = 10$ (dB).

Figure 3 presents the outage probabilities versus Ψ_4 (dB) when $\Psi_1 = 20(\text{dB}), \Psi_2 = 10(\text{dB})$, $\Psi_3 = 1(\text{dB}), \Psi_5 = 0.5(\text{dB}), \Psi_6 = -5(\text{dB}), \phi_1 = 0.3$, $\phi_2 = 0.7, \phi_3 = 0.4, \phi_4 = 0.6$, $N = 8$ and $Q = 10$ (dB). In Figure 3, in the proposed SIC-SC protocol, the outage probability of the secondary source SS_1 has a little reduction before is increased in the two cases of the perfect and imperfect SICs whereas the outage probability of the secondary source SS_2 is always increased because of variability between the interference channels and transmit powers. Furthermore, the system performance of the RRS comparison protocol in terms of the outage probabilities of the SS_1 and SS_2 is declined when the interference channel power Ψ_4 from the SS_1 to the PR occurs in the increasing sense. A reason for these results is that the transmit power allocation in the SIC-SC protocol is adjusted according to the interference channel powers Ψ_4, e.g. the transmit power of the SS_1 (denoted as P_{SS_1}) will be decreased when the Ψ_4 increase.

Figure 4 shows the outage probabilities of the SIC-SC and RRS protocols versus Ψ_1 (dB) in situations as $\Psi_2 = 10(\text{dB}), \Psi_3 = 1(\text{dB})$, $\Psi_4 = 1(\text{dB}), \Psi_5 = 0.5(\text{dB}), \phi_1 = 0.3, \phi_2 = 0.7, \phi_3 = 0.4$, $\phi_4 = 0.6$, $N = 8$, $Q = 10$ (dB) and $\Psi_6 \in \{-10(\text{dB}), -5(\text{dB})\}$. As shown in Figure 4, the system performance of the protocols SIC-SC and RRS is enhanced according the increasing of the channel power Ψ_1 because of high-success decoding of the first data in the SIC technology. In addition, the lower residual interference parameters lead to the higher performances of the secondary source SS_1. The case of the perfect SICs is viewed as $\Psi_6 \to -\infty$ (dB). In both protocols SIC-SC and RRS, the SIC and SC technologies improve the spectrum utilization efficiency by decreasing the number of the time slots from the secondary sources to the secondary relays and vice versus, but the SIC-SC protocol performs better by the adaptive power allocation parameters $\Psi_1 / (\Psi_1 + \Psi_2)$ and $\Psi_2 / (\Psi_1 + \Psi_2)$ in the coded s as a function of the channel powers Ψ_1.

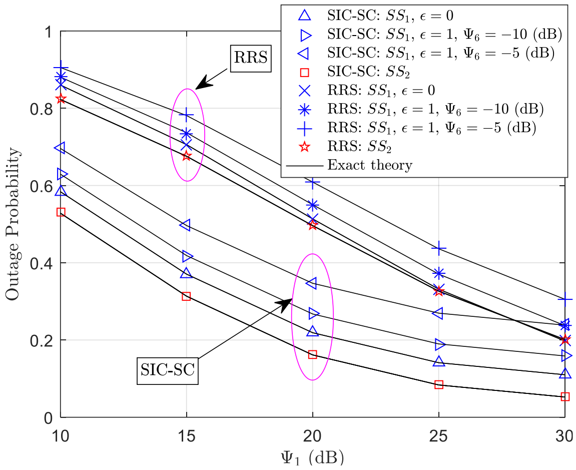

Figure 4. Outage probabilities of the SIC-SC and RRS protocols versus Ψ_1 (dB) when Ψ_2 = 10 (dB), Ψ_3 = 1 (dB), Ψ_4 = 1 (dB), Ψ_5 = 0.5 (dB), $\phi_1 = 0.3$, $\phi_2 = 0.7, \phi_3 = 0.4, \phi_4 = 0.6$, N = 8, Q = 10 (dB) and $\Psi_6 \in \{-10(\text{dB}), -5(\text{dB})\}$.

5. Conclusions

In this paper, we analyzed the underlay two-way DF scheme in which the secondary relays use the SIC technology to decode the data of two secondary sources sequentially, and then make a coded signal by the SC technology, known as the SIC-SC protocol. The SIC-SC protocol was designed to operate in two time slots under effects from the interference constraint of the primary receiver and residual interference of the imperfect SIC processes. Transmit powers provided to carry the data were allocated dynamically according to channel powers of interference and transmission. The secondary relay was selected from considering strongest channel gain subject to increase decoding capacity of the first data and decrease collection time of the CSIs. The identical underlay two-way DF operation with the random relay selection and the fixed power allocations (called the RRS protocol) was also investigated to compare the proposed SIC-SC protocol. The closed-form outage probability expressions were derived from mathematical manipulations and verified exactly by performing the Monte Carlo simulations. The simulation and analysis results shown that 1) the system performance of the proposed SIC-SC protocol in terms of the outage probabilities was enhanced by increasing the number of the secondary relays and the interference constraint as well as decreasing the residual interference powers, and 2) the non-identical outage performances of the secondary sources depended on both interference channel powers to the primary receiver and desired channel powers to the selected secondary relay from the secondary sources, and 3) the performance of the nearer secondary source is worse than that of the distant secondary source, and finally, the system performance of the proposed SIC-SC protocol outperforms that of the RRS protocol in terms of the outage probabilities.

Appendix A: Proof of Lemma 1

Substituting (7) into the probability of Δ_1 as in (9), the Δ_1 is expressed as

$$\Delta_1 = \Pr\left[\frac{1}{2}\log_2\left(1+\gamma_{SR_n\to S_1}\right) < R_{th}\right]$$
$$= \Pr\left[\gamma_{SR_n\to S_1} < 2^{2R_{th}}-1\right]$$
$$= \Pr\left[\frac{\Psi_5 Q g_{SS_1SR_n} g_{SS_2PR}}{\Psi_4 Q g_{SS_2SR_n} g_{SS_1PR} + \left(\Psi_4+\Psi_5\right) g_{SS_1PR} g_{SS_2PR}} < 2^{2R_{th}}-1\right]$$
$$= \Pr\left[\frac{g_{SS_1SR_n}}{g_{SS_1PR}} < \underbrace{\frac{\left(2^{2R_{th}}-1\right)\Psi_4}{\Psi_5}}_{\upsilon_1}\frac{g_{SS_2SR_n}}{g_{SS_2PR}} + \underbrace{\frac{\left(2^{2R_{th}}-1\right)\left(\Psi_4+\Psi_5\right)}{\Psi_5 Q}}_{\upsilon_2}\right] \qquad \text{(A.1)}$$
$$= \int_0^\infty f_{g_{SS_2SR_n}/g_{SS_2PR}}(x) F_{g_{SS_1SR_n}/g_{SS_1PR}}\left(\upsilon_1 x+\upsilon_2\right)dx,$$

where $f_{g_X/g_Y}(x)$ are $F_{g_X/g_Y}(x)$ are the PDF and CDF of the RVs g_X/g_Y, $X,Y \in \{\text{SS}_1,\text{SS}_2,\text{SR}_n,\text{PR}\}$, $n \in \{1,2,...,N\}$.

By referring from [29] (see equations (24–25)), the $f_{g_{SS_2SR_n}/g_{SS_2PR}}(x)$ is obtained as

$$f_{g_{SS_2SR_n}/g_{SS_2PR}}(x)=\frac{\partial F_{g_{SS_2SR_n}/g_{SS_2PR}}(x)}{\partial x}=\frac{\Psi_2\Psi_5}{(\Psi_2+\Psi_5x)^2}. \quad (A.2)$$

The $F_{g_{SS_1SR_n}/g_{SS_1PR}}(x)$ is expressed as

$$\begin{aligned}F_{g_{SS_1SR_n}/g_{SS_1PR}}(x)&=\Pr\left[\frac{g_{SS_1SR_n}}{g_{SS_1PR}}<x\right]\\&=\Pr\left[g_{SS_1SR_n}<xg_{SS_1PR}\right]\\&=\int_0^\infty f_{g_{SS_1PR}}(y)F_{g_{SS_1SR_n}}(xy)dy,\end{aligned} \quad (A.3)$$

where $F_{g_{SS_1SR_n}}(x)$ is the CDF of the RV $g_{SS_1SR_n}$ and is expressed as (see the equation (7–14) in [24])

$$F_{g_{SS_1SR_n}}(x)=\left(1-e^{-x/\Psi_1}\right)^N=\sum_{p=0}^{N}\binom{p}{N}(-1)^p e^{-px/\Psi_1}. \quad (A.4)$$

In (A.4), $\binom{p}{N}$ denotes the binomial coefficient $\binom{p}{N}=\frac{N!}{p!(N-p)!}$.

Then, the $F_{g_{SS_1SR_n}/g_{SS_1PR}}(x)$ is solved as

$$\begin{aligned}F_{g_{SS_1SR_n}/g_{SS_1PR}}(x)&=\int_0^\infty\frac{e^{-y/\Psi_4}}{\Psi_4}\sum_{p=0}^{N}\binom{p}{N}(-1)^p e^{-py/\Psi_1}dy\\&=\Psi_1\sum_{p=0}^{N}\binom{p}{N}\frac{(-1)^p}{\Psi_1+\Psi_4px}.\end{aligned} \quad (A.5)$$

Substituting (A.2) and (A.5) into (A.1), the Δ_1 is manipulated equivalently as

$$\begin{aligned}\Delta_1&=\int_0^\infty\frac{\Psi_2\Psi_5}{(\Psi_2+\Psi_5x)^2}\Psi_1\sum_{p=0}^{N}\binom{p}{N}\frac{(-1)^p}{\Psi_1+\Psi_4p(\upsilon_1x+\upsilon_2)}dx\\&=\Psi_1\Psi_2\Psi_5\sum_{p=0}^{N}\binom{p}{N}\int_0^\infty\frac{(-1)^p dx}{(\Psi_1+p\Psi_4\upsilon_2+p\Psi_4\upsilon_1x)(\Psi_2+\Psi_5x)^2}.\end{aligned} \quad (A.6)$$

By performing variable transformations as $y=(\Psi_2+\Psi_5x)^{-1}$ and $x=\upsilon_4(p)+\frac{(\Psi_5\upsilon_3(p)-\Psi_2\upsilon_4(p))y}{\Psi_2\Psi_5}$, where $\upsilon_3(p)=(\Psi_1+p\Psi_4\upsilon_2)/(\Psi_1\Psi_4)$ and $\upsilon_4(p)=p\upsilon_1/\Psi_1$, the Lemma 1 is proven completely.

Appendix B: Proof of Lemma 2

From (8) and (A.1), the probability Δ_2 as in (9) is expressed as

$$\begin{aligned}\Delta_2&=\Pr\left[\begin{array}{l}\left(\begin{array}{l}\frac{\Psi_5Qg_{SS_1SR_n}g_{SS_2PR}}{\Psi_4Qg_{SS_2SR_n}g_{SS_1PR}+(\Psi_4+\Psi_5)g_{SS_1PR}g_{SS_2PR}}\\ \geq 2^{2R_{th}}-1\end{array}\right)\\ \cap\left(\frac{1}{2}\log_2\left(1+\gamma_{SR_n\to s_2}\right)<R_{th}\right)\end{array}\right]\\&=\Pr\left[\begin{array}{l}\left(\frac{g_{SS_1SR_n}}{g_{SS_1PR}}\geq\frac{\upsilon_1g_{SS_2SR_n}}{g_{SS_2PR}}+\upsilon_2\right)\\ \cap\left(\frac{P_{SS_2}g_{SS_2SR_n}}{\varepsilon I|h_n|^2+N_0}<2^{2R_{th}}-1\right)\end{array}\right]\\&=\Pr\left[\begin{array}{l}\left(\frac{g_{SS_1SR_n}}{g_{SS_1PR}}\geq\frac{\upsilon_1g_{SS_2SR_n}}{g_{SS_2PR}}+\upsilon_2\right)\cap\\ \left(\begin{array}{l}\frac{g_{SS_2SR_n}}{g_{SS_2PR}}<\underbrace{(\Psi_4+\Psi_5)\left(2^{2R_{th}}-1\right)/(\Psi_4Q)}_{\upsilon_5}\\ +\varepsilon(\Psi_4+\Psi_5)\left(2^{2R_{th}}-1\right)g_n/\Psi_4\end{array}\right)\end{array}\right].\end{aligned} \quad (B.1)$$

To solve the Δ_2 in (B.1) by closed-form expressions, we consider two cases of perfect SICs $(\varepsilon=0)$ and imperfect SICs $(\varepsilon=1)$ as follows.

- Perfect SICs ($\varepsilon=0$): By referring from (A.2) and using (A.5), the Δ_2 is obtained as

$$\begin{aligned}\Delta_2&=\int_0^{\upsilon_5}f_{g_{SS_2SR_n}/g_{SS_2PR}}(x)\left(1-F_{g_{SS_1SR_n}/g_{SS_1PR}}(\upsilon_1x+\upsilon_2)\right)dx\\&=F_{g_{SS_2SR_n}/g_{SS_2PR}}(\upsilon_5)\\&\quad-\int_0^{\upsilon_5}f_{g_{SS_2SR_n}/g_{SS_2PR}}(x)F_{g_{SS_1SR_n}/g_{SS_1PR}}(\upsilon_1x+\upsilon_2)dx\\&=\frac{\Psi_5\upsilon_5}{\Psi_2+\Psi_5\upsilon_5}-\Psi_1\Psi_2\Psi_5\sum_{p=0}^{N}\binom{p}{N}(-1)^p\\&\quad\times\int_0^{\upsilon_5}\frac{dx}{(\Psi_1+p\Psi_4\upsilon_2+p\Psi_4\upsilon_1x)(\Psi_2\lambda_5+\Psi_5\lambda_2x)^2}.\end{aligned} \quad (B.2)$$

Also performing as (A.6), Δ_2 is solved as in Lemma 2 with the case $\varepsilon=0$.

- Imperfect SICs ($\varepsilon=1$): The Δ_2 in this case ($\varepsilon=1$) is presented as

$$\Delta_2 = \int_0^\infty f_{g_n}(x) \int_0^{\upsilon_5(1+Qx)} \left(1 - F_{g_{SS_1SR_n}/g_{SS_1PR}}(\upsilon_1 y + \upsilon_2)\right) \times f_{g_{SS_2SR_n}/g_{SS_2PR}}(y) dy dx$$
$$= \int_0^\infty \frac{e^{-x/\Psi_6}}{\Psi_6} \left\{ \frac{\Psi_5\upsilon_5(1+Qx)}{\Psi_2 + \Psi_5\upsilon_5(1+Qx)} - \frac{1}{\Psi_4}\sum_{p=0}^{N}\binom{p}{N}\frac{(-1)^p}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)} \times \left(\begin{aligned} &\frac{\upsilon_5(1+Qx)\Psi_5^2}{\Psi_2 + \Psi_5\upsilon_5(1+Qx)} - \frac{\Psi_2\Psi_5\upsilon_4(p)}{\Psi_5\upsilon_3(p) - \Psi_2\upsilon_4(p)} \\ &\times \ln\left(\frac{(\Psi_2 + \Psi_5\upsilon_5(1+Qx))\upsilon_3(p)}{\Psi_2(\upsilon_3(p) + \upsilon_5(1+Qx)\upsilon_4(p))} \right) \end{aligned} \right) \right\} dx. \quad \text{(B.3)}$$

By extending $\ln(\cdot)$, changing varibables and then solving the integrals in (B.3), the probability Δ_2 is answered as in Lemma 2 with the remaining case $\varepsilon = 1$. Hence, the Lemma 2 is verified commpletely.

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References

[1] P. Popovski, H. Yomo, Physical Network Coding in Two-Way Wireless Relay Channels, presented at 2007 IEEE International Conference on Communications (ICC), Glasgow, 2007, 707-712. https://doi.org/10.1109/ICC.2007.121.

[2] Z. Cao, X. Ji, J. Wang, S. Zhang, Y. Ji, J. Wang, Security-Reliability Tradeoff Analysis for Underlay Cognitive Two-Way Relay Networks, IEEE Transactions on Wireless Communications 18(12) (2019) 6030-6042. https://doi.org/10.1109/TWC.2019.2941944.

[3] J. Mitola, G.Q. Maguire, Cognitive radio: making software radios more personal, IEEE Personal Communications 6(4) (1999) 13-18. https://doi. org /10.1109/98.788210.

[4] T.M.C. Chu, H. Zepernick, Performance Optimization for Hybrid Two-Way Cognitive Cooperative Radio Networks With Imperfect Spectrum Sensing, IEEE Access 6 (2018) 70582-70596. https://doi.org/10.1109/ICC.2007.121.

[5] K. Ho-Van, T. Do-Dac, Security Analysis for Underlay Cognitive Network with Energy-Scavenging Capable Relay over Nakagami-m Fading Channels, Wireless Communications and Mobile Computing 2019 1-16. https://doi.org/ 10.1155/2019/5080952.

[6] X. Zhang, Z. Zhang, J. Xing, R. Yu, P. Zhang, W. Wang, Exact Outage Analysis in Cognitive Two-Way Relay Networks With Opportunistic Relay Selection Under Primary User's Interference, IEEE Transactions on Vehicular Technology 64(6) (2015) 2502-2511. https://doi.org/10.1109/ TVT.2014.2346615.

[7] T.T. Duy, H.Y. Kong, Exact outage probability of cognitive two-way relaying scheme with opportunistic relay selection under interference constraint, IET Communications 6(16) (2012), 2750-2759. https://doi.org/ 10.1049/iet-com. 2012.0235.

[8] H.V. Toan, V.N.Q. Bao, Opportunistic relaying for cognitive two-way network with multiple primary receivers over Nakagami-m fading, presented at 2016 International Conference on Advanced Technologies for Communications (ATC), Hanoi city, 2016, 141-146. https://doi.org/10.1109/ATC.2016.7764762.

[9] H.V. Toan, V.N.Q. Bao, H. Nguyen-Le, Cognitive two-way relay systems with multiple primary receivers: exact and asymptotic outage formulation, IET Communications 11(16) (2017), 2490-2497. https://doi.org/10.1049/iet-com.2017. 0400.

[10] H.V.Toan, V.N.Q. Bao, K.N. Le, Performance analysis of cognitive underlay two-way relay networks with interference and imperfect channel state information, EURASIP Journal on Wireless Communications and Networking 2018 53 (2018). https://doi.org/10.1186/s13638-018-1063-z.

[11] S. Solanki, P.K. Sharma, P.K. Upadhyay, Adaptive Link Utilization in Two-Way Spectrum Sharing Relay Systems Under Average Interference Constraints, IEEE Systems Journal 12(4) (2018), 3461-3472. https://doi.org/10.1109/ JSYST.2017.2713887.

[12] X. Yue, Y. Liu, S. Kang, A. Nallanathan, Y. Chen, Modeling and Analysis of Two-Way Relay Non-Orthogonal Multiple Access Systems, IEEE Transactions on Communications 66(9) (2018), 3784-3796. https://doi.org/10.1109/TCOMM. 2018.2816063.

[13] X. Zou, B. He, H. Jafarkhani, An Analysis of TwoUser Uplink Asynchronous Non-orthogonal Multiple Access Systems, IEEE Transactions on Wireless Communications 18(2) (2019), 1404-1418. https://doi.org/10.1109/TWC.2019.2892486

[14] Z. Yang, Z. Ding, P. Fan, N. Al-Dhahir, The Impact of Power Allocation on Cooperative Nonorthogonal Multiple Access Networks With SWIPT, IEEE Transactions on Wireless Communications 16(7) (2017), 4332-4343. https://doi.org/10.1109/TWC.2017.2697380.

[15] P.N. Son, T.T. Duy, K. Ho-Van, SIC-Coding Schemes for Underlay Two-Way Relaying Cognitive Networks, Wireless Communications and Mobile Computing 2020 1-24. https://doi.org/ 10.1155/2020/8860551.

[16] M.F. Kader, M.B. Shahab, S.Y. Shin, Exploiting Non-Orthogonal Multiple Access in Cooperative Relay Sharing, IEEE Communications Letters 21(5) (2017) 1159-1162. https://doi.org/ 10.1109/LCOMM.2017.2653777.

[17] X. Yue, Y. Liu, S. Kang, A. Nallanathan, Z. Ding, Spatially Random Relay Selection for Full/Half-Duplex Cooperative NOMA Networks, IEEE Transactions on Communications 66(8) (2018) 3294-3308. https://doi.org/10.1109/TCOMM. 2018.2809740.

[18] Y. Liu, Z. Ding, M. Elkashlan, J. Yuan, Nonorthogonal Multiple Access in Large-Scale Underlay Cognitive Radio Networks, IEEE Transactions on Vehicular Technology 65(12) (2016) 10152-10157. https://doi.org/10.1109/ TVT.2016.2524694.

[19] Y. Song, W. Yang, Z. Xiang, N. Sha, H. Wang, Y. Yang, An Analysis on Secure Millimeter Wave NOMA Communications in Cognitive Radio Networks, IEEE Access 8 (2020), 78965-78978. https://doi.org/10.1109/ACCESS.2020.2989860.

[20] X. Ding, T. Song, Y. Zou, X. Chen, L. Hanzo, Security-Reliability Tradeoff Analysis of Artificial Noise Aided Two-Way Opportunistic Relay Selection, IEEE Transactions on Vehicular Technology 66(5) (2017) 3930-3941. https://doi.org/10.1109/TVT.2016.2601112.

[21] B. Zheng, M. Wen, F. Chen, J. Tang, F. Ji, Secure NOMA Based Full-Duplex Two-Way Relay Networks with Artificial Noise against Eavesdropping, presented at 2018 IEEE International Conference on Communications (ICC), Kansas City, 2018, 1-6. https://doi.org/ 10.1109/ICC.2018.8422946.

[22] P.N. Son, H.Y. Kong, Exact Outage Analysis of Energy Harvesting Underlay Cooperative Cognitive Networks, IEICE Transactions on Communications E98-B(4) (2015), 661-672. https://doi.org/10.1587/transcom.E98.B.661.

[23] K. Tourki, K.A. Qaraqe, M. Alouini, Outage Analysis for Underlay Cognitive Networks Using Incremental Regenerative Relaying, IEEE Transactions on Vehicular Technology 62(2) (2013) 721-734. https://doi.org/10.1109/TVT. 2012.2222947.

[24] A. Papoulis, S.U. Pillai, Probability, random variables and stochastic processes, 4th ed., McGrawHill, New York, 2002.

[25] L. Pei, T. Zhifeng, L. Zinan, E. Erkip, S. Panwar, Cooperative wireless communications: a cross-layer approach, IEEE Wireless Communications 13(4) (2006) 84-92. https://doi.org/10.1109/ MWC.2006.1678169.

[26] A. Ghasemi, E.S. Sousa, Fundamental limits of spectrum-sharing in fading environments, IEEE Transactions on Wireless Communications 6(2) (2007) 649-658. https://doi.org/10.1109/TWC. 2007.05447.

[27] J. M. Peha, Approaches to spectrum sharing, IEEE Communications Magazine 43(2) (2005) 10-12. https://doi.org/10.1109/MCOM.2005. 1391490.

[28] H. Kim, S. Lim, H. Wang, D. Hong, Optimal Power Allocation and Outage Analysis for Cognitive Full Duplex Relay Systems, IEEE Transactions on Wireless Communications 11(10) (2012) 3754-3765. https://doi.org/10.1109/TWC. 2012.083112.120127.

[29] P.N. Son, T.T. Duy, Performance analysis of underlay cooperative cognitive full-duplex networks with energy-harvesting relay, Computer Communications 122 (2018) 9-19. https://doi.org/ 10.1016/j.comcom.2018.03.003.

[30] T.V. Nguyen, T. Do, V.N.Q. Bao, D.B.d. Costa, B. An, On the Performance of Multihop Cognitive Wireless Powered D2D Communications in WSNs, IEEE Transactions on Vehicular Technology 69(3) (2020) 2684-2699. https://doi.org/10.1109/TVT.2020.2963841.

[31] Y. Ruan, Y. Li, C. Wang, R. Zhang, H. Zhang, Energy Efficient Power Allocation for Delay Constrained Cognitive Satellite Terrestrial Networks Under Interference Constraints, IEEE Transactions on Wireless Communications 18(10) (2019) 4957-4969. https://doi.org/10.1109/TWC. 2019.2931321.

[32] H. Gao, S. Zhang, Y. Su, M. Diao, M. Jo, Joint Multiple Relay Selection and Time Slot Allocation Algorithm for the EH-Abled Cognitive Multi-User Relay Networks, IEEE Access 7 (2019) 111993-112007. https://doi.org/10.1109/ ACCESS.2019.2932955.

[33] H. Arezumand, H. Zamiri-Jafarian, E. Soleimani-Nasab, Exact and Asymptotic Analysis of Partial Relay Selection for Cognitive RF-FSO Systems With Non-Zero Boresight Pointing Errors, IEEE Access 7 (2019) 58611-58625. https://doi.org/10.1109/ACCESS.2019.2914480.

[34] P.N. Son, H.Y. Kong, Energy-Harvesting Relay Selection Schemes for Decode-and-Forward Dual-Hop Networks, IEICE TRANSACTIONS on Communications E98-B(12) (2015) 2485-2495. https://doi.org/10.1587/transcom.E98.B.2485.

[35] T.N. Nguyen, T.H. Quang Minh, P.T. Tran, M. Voznak, T.T. Duy, T.-L. Nguyen, P.T. Tin, Performance enhancement for energy harvesting based two-way relay protocols in wireless ad-hoc networks with partial and full relay selection methods, Ad Hoc Networks 84 (2019) 178-187. https://doi.org/10.1016/j.adhoc.2018.10.005.

[36] L. Pan, Z. Li, Z. Wang, F. Zhang, Joint Relay Selection and Power Allocation for the Physical Layer Security of Two-Way Cooperative Relaying Networks, Wireless Communications and Mobile Computing 2019 1-7. https://doi.org/10.1155/2019/1839256.

[37] A.A. Nasir, Z. Xiangyun, S. Durrani, R.A. Kennedy, Relaying Protocols for Wireless Energy Harvesting and Information Processing, IEEE Transactions on Wireless Communications 12(7) (2013) 3622-3636. https://doi.org/10.1109/TWC. 2013.062413.122042.

[38] R.I. Gradshteyn, I.M. Ryzhik, A.Jeffrey, D. Zwillinger, Table of integral, series and products, 7th ed., Elsevier, Amsterdam, 2007.

[39] H. Haiyan, L. Zan, S. Jiangbo, G. Lei, Underlay cognitive relay networks with imperfect channel state information and multiple primary receivers, IET Communications 9(4) (2015) 460-467. https://doi.org/10.1049/iet-com.2014.0429.

[40] B. Zhong, Z. Zhang, Opportunistic Two-Way Full-Duplex Relay Selection in Underlay Cognitive Networks, IEEE Systems Journal 12(1) (2018) 725-734. https://doi.org/10.1109/JSYST. 2016.2514601.