Combining Power Allocation and Superposition Coding for an Underlay Two-way Decode-and-forward Scheme

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Abstract: In this paper, we analyze an underlay two-way decode-and-forward scheme in which secondary relays use successive interference cancellation (SIC) technology to decode data of two secondary sources sequentially, and then generate a coded signal by superposition coding (SC) technology, denoted as SIC-SC protocol. The SIC-SC protocol is designed to operate in two time slots under effects from an interference constraint of a primary receiver and residual interference of imperfect SIC processes. Transmit powers provided to carry the data are allocated dynamically according to channel powers of interference and transmission, and a secondary relay is selected from considering strongest channel gain subject to increase in decoding capacity of the first data and decrease in collection time of channel state information. Closed-form outage probability expressions are derived from mathematical manipulations and verified by performing Monte Carlo simulations. An identical scheme of underlay two-way decode-and-forward relaying with random relay selection and fixed power allocations is considered to compare with the proposed SIC-SC protocol, denoted as RRS protocol. Simulation and analysis results show that the non-identical outage performances of the secondary sources in the proposed SIC-SC protocol are improved by increasing the number of the secondary relays and the interference constraint as well as decreasing the residual interference powers. Secondly, the performance of the nearer secondary source is worse than that of the farther secondary source. In addition, the proposed SIC-SC protocol outperforms the RRS comparison protocol, and effect of power allocations through channel powers is discovered. Finally, derived theory values are precise to simulation results.

Keywords: Successive interference cancellation, superposition coding, power allocation, underlay cognitive radio, non-orthogonal multiple access, outage probability.

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1. Introduction

Two-way communication is a protocol for exchanging data back and forth between users in which the users are both receiver and transmitter points [1, 2]. Energy and spectrum utilization efficiencies have been enhanced by two-way cooperation [1, 2]. Intermediate relays operating as amplify-and-forward (AF) and decode-and-forward (DF) devices process received signals before forwarding back to the users. With lower transmit powers, the noise-added data amplified is always sent to the desired users in the AF operation whereas the cooperative relays in the DF operation drop those data due to unsuccessful decoding. However, with higher transmit powers, the noises are cleared by the DF relays and then the desired users get a high success rate of decoding. Transmit frequency spectrum of the users and relays can be licensed or are shared by primary users (PUs) [3-5]. Spectrum sharing solution known as cognitive radio is to enhance bandwidth demand for mobile multimedia services and the explosive development of next-generation wireless networks such as wireless sensor networks, Internet-of-things (IoT), next-generation mobile networks,... where secondary users (SUs) collaborate with the PUs [3-5]. In the cognitive radio networks, the SUs can transmit at any time so that the interference powers at the PUs are limited under tolerable thresholds, denoted as underlay operation protocol [4, 5]. The tolerable thresholds are gifts of the primary networks sent to the secondary networks based on the quality-of-service (QoS) contracts. As a result, the transmit power of the SUs must be adjusted continuously as a function of tolerable interference thresholds and channel gains.

Investigations on performance of underlay two-way relaying systems have been considered in [7-11]. System performance of secondary two-way networks under an interference constraint of a primary receiver was improved by combining digital network coding and opportunity relay selection in which a selected secondary relay created a new data by XOR operation after decoding received data successfully [7]. Effects of multiple primary receivers have been investigated in [8, 9]. The authors in [10] took imperfect channel state information (CSI) from the SUs to the Pus into probability analyses. S. Solanki et. al in [11] exploited direct links to build adaptive protocols under average interference constraints. The transmit powers of the secondary sources and the secondary relays were set independently to maximum of achievable thresholds [7, 10, 11] or minimum of internal powers and mutual interference constraints [8, 9]. In addition, most of these investigations have been proposed on three-phase solutions, and therefore the bandwidth utilization is divided by three times.

The authors in [12] employed superposition coding (SC) at each source group and successive interference cancellation (SIC) at a relay to send lots of broadcast data whereas only using two time slots (two phases) in two-way relaying networks. The SC and SIC are core technologies in nonorthogonal multiple access (NOMA) systems [13]. The SC technology is used at the transmit source to merge signals with different powers based on distances to the destinations. The farther destinations are allocated by the higher signal powers. The nearby destinations apply the SIC technology to decode the derided signals by canceling the higher-powered signals and treating the lower-powered signals as interference [14]. However, decoding operations by the SIC technology can be imperfect because of residual interference signals [12].

Combination of the SC technology and the power allocation in the NOMA networks has been investigated in [12, 14-19]. Transmit
powers under control of interference constraint [15, 18, 19] and maximum power limitation [12, 14-17] were allocated to fixed values to share a part of the total power to different users in multiple access operations. In [18], transmit power of a base station in multicast networks was achieved to maximum constraint following a min-max rule of overall maximum permissible interference powers and maximum transmission power whereas power allocation coefficient were constant values based on channel gains from the base station to multi users. A similar set-up model as [18] has also been considered in a recent study [19] to against eavesdroppers. Power allocation coefficients in the SC technology without constant values should be considered in multiple-access networks.

In this paper, we analyze an underlay two-way DF scheme with two secondary sources, multiple secondary relays and a primary receiver. In this scheme, the secondary relays use the SIC technology to decode the data of two secondary sources sequentially, and then generate a coded signal by the SC technology, denoted as SIC-SC protocol. By applying the uplink NOMA protocol, the SIC technology and the SC technology, the SICSC protocol operates in two time slots, and also suffers an interference constraint of the primary receiver and residual interference of the imperfect SIC processes. Transmit powers provided to carry the data are allocated dynamically according to channel powers of interference and transmission. A secondary relay is selected from considering strongest channel gain subject to increase in decoding capacity of the first data and decrease in collection time of CSIs. System performance of the SIC-SC protocol is evaluated by closed-form outage probability expressions. These outage probability analyses are verified by performing Monte Carlo simulations. An identical scheme of underlay two-way DF relaying with random relay selection and the

fixed power allocations are considered to compare the proposed SIC-SC protocol, denoted as RRS protocol. The simulation and analysis results show contributions as follows. Firstly, with the SIC and SC technologies combining the power allocations, the non-identical outage performances of the secondary sources are improved when the number of the secondary relays and the interference constraint are increased as well as the residual interference powers are controlled to decrease. Secondly, the performance of the nearer secondary source is worse than that of the farther secondary source. Thirdly, the proposed SIC-SC protocol outperforms the RRS comparison protocol in terms of the outage probabilities, and discussions on effect of power allocations through channel powers are presented. Finally, derived theory values are precise to simulation results.

This paper is organized as follows. Section 2 presents a system model of an underlay two-way DF scheme. Section 3 analyzes outage probabilities of the proposed SIC-SC protocol and the RRS comparison protocol. Analysis and simulation results are presented in Section 4. Finally, Section 5 summarizes contributions.

2. System Model

Figure 1 presents a system model of an underlay two-way DF scheme in which two secondary sources SS1 and SS2 send corresponding data s1 and s2 to each other with the help of a closed group of N intermediate secondary relays SRi, where i ∈ {1,2,...,N}. The secondary network nodes SS1, SS2 and SR have identical variances of the zero-mean white Gaussian noises (AWGN) (denoted as N0), and are in an interference constraint of a primary receiver PR (denoted as I). The secondary relays SRi are nearer to the secondary source SS1 than the SS2, and thus the secondary relays SRi use the SIC technology to decode the data
s\textsubscript{i} firstly. In Figure 1, a direct transmission between secondary sources SS\textsubscript{1} and SS\textsubscript{2} is skipped by far distance or deep shadow fading [2, 20], and secondary and primary nodes are installed with a single antenna.

Figure 1. System model.

In Figure 1, \(h_{XY}\) denotes wireless channels of links \(X - Y\) which are modeled as complex normal distributions \(h_{XY} \sim CN(0, \Psi_{XY})\) with zero means and normalized variances \(\Psi_{XY}\) (\(\Psi_{XY}\) are also the normalized powers of the channels) [21], where \(X,Y \in \{SS_1, SS_2, SR, PR\}\).

For the reason that the secondary relays SR\textsubscript{i} are located in the closed cluster and are closer to SS\textsubscript{1} than SS\textsubscript{2}, the channel variances are set as \(\Psi_{SS_iSR} = \Psi_{SS_iPR} = \Psi_1\), \(\Psi_{SS_iSR} = \Psi_{SS_iSS} = \Psi_2\), \(\Psi_{SR_{PR}} = \Psi_3\), \(\Psi_{SS_iPR} = \Psi_4\), \(\Psi_{SS_iSR} = \Psi_5\) and \(\Psi_1 > \Psi_2\) [22, 23]. As a result, \(g_{XY} = \left|h_{XY}\right|^2\) are exponentially distributed random variables (RVs) with the probability density function (PDF) as \(f_{g_{XY}} (x) = e^{-x/\Psi_{XY}} / \Psi_{XY}\) and the cumulative distribution function (CDF) as \(F_{g_{XY}} (x) = 1 - e^{-x/\Psi_{XY}}\) [24] (see the equation (6–68)).

Traditionally, a set-up phase is performed according to the cooperative medium access control protocol [25]. The secondary nodes SS\textsubscript{i}, SS\textsubscript{2} and SR\textsubscript{i} can know perfectly the CSI\textsubscript{s} \(h_{SS_iPR}\), \(h_{SS_iSR}\) and \(h_{SR_iPR}\) respectively by directly feedback channels from the primary receiver PR [22, 26] or by indirectly feedback channels from a third party [22, 27], \(i \in \{1, 2, \ldots, N\}\). Firstly, the secondary sources SS\textsubscript{1} and SS\textsubscript{2} broadcast request-to-send messages (RTS) sequentially to the secondary relays SR\textsubscript{i} with small transmit powers and low rates. The RTS messages contain its information such as the links to the PR. From receiving these RTS messages, the SR\textsubscript{i} can estimate the fading channels \(h_{SS_iSR}\) and \(h_{SS_iSR}\), and then broadcast a helper-ready-to-send (HTS) message, which contains the \(h_{SS_iSR}\) and \(h_{SR_iPR}\), to the SS\textsubscript{1} and SS\textsubscript{2}. Next, the SS\textsubscript{1} and SS\textsubscript{2} can estimate the fading channels \(h_{SR_iSS_i}\) and \(h_{SR_iSS_i}\) from the received HTS messages, respectively. Therefore, the SS\textsubscript{1} can know perfectly all fading channels to the secondary relays and the information link of the SS\textsubscript{2} to allocate the transmit powers and select the cooperative secondary relay. In addition, the selected relay can use the detected and estimated information to cancel interference. Finally, the SS\textsubscript{1} will send a clear-to-send message (CTS) including initial parameters to begin a next data transmission phase.

In the underlay cognitive radio schemes, the interference at the primary receiver PR from the secondary network are less than or equal the constraint \(I\) [28, 29]. Inequalities related to transmit powers and channel gains are obtained as

\[
\begin{align*}
P_{SS_i} g_{SS_iPR} + P_{SS_i} g_{SS_iSR} & \leq I \\
P_{SR_i} g_{SR_iPR} & \leq I
\end{align*}
\]

where \(P_{SS_i}\), \(P_{SS_i}\) and \(P_{SR_i}\) are transmit powers of the SS\textsubscript{1}, the SS\textsubscript{2} and the SR\textsubscript{i}, respectively.

To maximize system performance of the secondary network, and coordinate balance manner between the transmit powers and channel gains from secondary sources SS\textsubscript{1} and
SS2 to the PR, we allocate the transmit powers \( P_{SS1} \), \( P_{SS2} \), and \( P_{SR} \) as \( P_{SS1} = I\Psi_{1}/(g_{SS,PR} (\Psi_{4} + \Psi_{5})) \), \( P_{SS2} = I\Psi_{4}/(g_{SS,PR} (\Psi_{4} + \Psi_{5})) \) and \( P_{SR} = I/g_{SR,PR} \). It is worth noting that this allocation proposal is used in many published literature [7, 23, 30-33].

The operation principle of the SIC-SC protocol occurs in two time slots and is presented by mathematical models as follows. In the first time slot, the secondary sources SS1 and SS2 send the data \( s_1 \) and \( s_2 \), respectively, with the same carrier frequency to the secondary relays SRs, at the same time, where \( i \in \{1, 2, ..., N\} \). The received signal at the secondary relays SRs is stated as

\[
y_{SR} = \sqrt{P_{SS1}} s_1 h_{SR} + \sqrt{P_{SS2}} s_2 h_{SR} + n_{SR},
\]

(2)

where \( E\{s_1^2\} = E\{s_2^2\} = 1 \) and \( n_{SR} \) denote the AWGNs at the SRs with the same normalized variance \( N_0 \) ( \( E\{\cdot\} \) denote the expectation operator).

Since \( \Psi_{1} > \Psi_{4} \) (SR is closer to the SS1 than the SS2), based on the SIC technology, the SRi will decode the data \( s_1 \) in (2) firstly by considering the signal \( \sqrt{P_{SS1}} s_1 h_{SR} \) as interference. From (2), the received signal-to-interference-plus-noise ratios (SINRs) at SRi to decode \( s_1 \) is expressed as

\[
\gamma_{SR_i \rightarrow s_1} = \frac{P_{SS1} g_{SS,SR}}{P_{SS2} g_{SS,SR} + N_0} = \frac{\Psi_{1} Q g_{SS,SR} g_{SS,PR} + (\Psi_{4} + \Psi_{5}) g_{SS,PR} g_{SS,PR}}{\Psi_{4} Q g_{SS,SR} g_{SS,PR}},
\]

(3)

where \( Q = I/N_0 \).

From acquisitions of perfect CSIs, the secondary source SS1 only select one secondary relay (denoted as \( SR_n \), \( n \in \{1, 2, ..., N\} \)) so that the decoding capacity is increased and the number of the pilot channels is decreased. The secondary relay \( SR_n \) is obtained as \( SR_n = \arg \max_{i=1, 2, ..., N} g_{SS,SR} \). As a result, the SINR at the SRn in (3) can achieve higher value when comparing with relay selection randomly (take any secondary relay in a group of \( N \) secondary relays).

By the SIC procedure completely or partly, the interference part \( \sqrt{P_{SS1}} s_1 h_{SS,SR} \) in (2) can be canceled after the data \( s_1 \) is decoded successfully, and thus, the received signal at the secondary relays \( SR_n \) after the SIC procedure is expressed as

\[
y_{SR_n \rightarrow s_2} = y_{SR_n} - \sqrt{P_{SS1}} s_1 h_{SS,SR} = \sqrt{P_{SS2}} s_2 h_{SS,SR} + \sqrt{\epsilon I} h_n + n_{SR_n},
\]

(4)

where \( \sqrt{\epsilon I} \) is a residual interference component at the SRn due to imperfect SIC procedure; \( \epsilon = 0 \) and \( \epsilon = 1 \) express perfect and imperfect interference cancellation at the SRn, respectively; \( h_n \) is modeled as an identical complex normal distribution \( h_n \sim CN(0, \Psi_6) \) [12, 16] with zero mean and same normalized variance \( \Psi_6 \), and thus \( g_n = |h_n|^2 \) are also exponentially distributed RVs with the PDF as \( f_{g_n}(x) = e^{-x/\Psi_6}/\Psi_6 \) and the CDF as \( F_{g_n}(x) = 1 - e^{-x/\Psi_6} \) [24] (see the equations (6-68)).

The received SINR at the selected secondary relay \( SR_n \) to decode \( s_2 \) is obtained and manipulated as

\[
\gamma_{SR_n \rightarrow s_2} = \frac{\Psi_{4} Q g_{SS,SR} g_{SS,PR}}{\epsilon Q g_{n} + 1}(\Psi_{4} + \Psi_{5}).
\]

(5)

If the selected secondary relay \( SR_n \) decodes successfully both data \( s_1 \) and \( s_2 \), a coded data \( s \) is created by the SC technology as

\[
s = s_1 \sqrt{\Psi_{1}/(\Psi_{1} + \Psi_{2})} + s_2 \sqrt{\Psi_{2}/(\Psi_{1} + \Psi_{2})}.
\]

In this case, because the \( SR_n \) is nearer to the secondary source SS1 than to the secondary source SS2, thus, the \( SR_n \) sends the data \( s_2 \) to the
SS₁ with the smaller power parameter \( \Psi_2/(\Psi_1 + \Psi_2) \) and vice versa.

In the second time slot, the SRₐ broadcasts back the coded data s to two secondary sources SS₁ and SS₂, and then the received signals at the secondary sources SSₓ are obtained as:

\[
y_{SS} = \sqrt{P_{SR_x} s h_{SR,SS_x}} + n_{SS_x}
\]

\[
= \sqrt{P_{SR_x} s (\Psi_1 + \Psi_2)} h_{SR,SS_x} + n_{SS_x} + \sqrt{P_{SR_x} s (\Psi_1 + \Psi_2)} s h_{SR,SS_x} + n_{SS_x},
\]

where \( n_{SS_x} \) denote AWGNs at the SSₓ with the same normalized variance \( N_0 \), and \( k \in \{1, 2\} \).

We assume that the SS₁ can perfectly cancel the known components including its data \( s_k \), i.e.,

\[
\sqrt{P_{SR_x} s (\Psi_1 + \Psi_2)} s h_{SR,SS_x} \quad \text{in (6).}
\]

The received SINRs at the secondary sources SSₓ are obtained to take \( s_l \), where \( l \in \{1, 2\} \) and \( l \neq k \), as

\[
y_{SS} = \frac{P_{SR_x} s h_{SR,SS_x}}{N_0 (\Psi_1 + \Psi_2)} = \frac{Q g_{SR,SS_x} \Psi_j}{g_{SR,PR} (\Psi_1 + \Psi_2)}. \quad (7)
\]

Here, we received data rates at the SS₁, SS₂ and SRₓ as

\[
R_{Z\rightarrow z} = \frac{1}{2} \log_2 (1 + \gamma_{Z\rightarrow z}) \text{(bits/s/Hz)}, \quad (8)
\]

where \( \frac{1}{2} \) shows that the proposed SIC-SC protocol operates in the two time slots, \( Z \in \{SS₁, SS₂, SRₓ\} \), \( z \in \{s₁, s₂\} \) and \( n \in \{1, 2, ..., N\} \).

For comparison purpose, we also consider a random relay selection (RRS) protocol with fixed power allocations where a collaborative SR, is randomly selected in a group of \( N \) secondary relays for the two-way relaying, \( i \in \{1, 2, ..., N\} \) [17, 34-36]. In particular, from (1), the transmit powers of the SS₁ and SS₂ are maximally set as \( P^{RBS}_{SS₁} = \phi₁/\gamma_{SS₁PR} \) and \( P^{RBS}_{SS₂} = \phi₂/\gamma_{SS₂PR} \) where \( \phi₁ + \phi₂ = 1 \) [15]. In addition, the secondary relay SRₓ applies the SC technology with power allocation parameters \( \phi_3 \) and \( \phi₄ \) to create a coded data as \( s_{RBS} = s_2 \sqrt{\phi_3} + s₁ \sqrt{\phi₄} \), where \( \phi_3 + \phi₄ = 1 \) and \( \phi₃ ≤ \phi₄ \) (the SS₁ is the nearer user to receive the data \( s₂ \)) [14].

3. Outage Probability Analyses

Outage probability at a node \( \gamma \) is defined as probability that the node \( \gamma \) cannot decode successfully the desired data or the received data rate at the node \( \gamma \) is less than a threshold data rate \( R_{th} \) [29,37] \( R_{Y\rightarrow z} \leq R_{th}, \quad \gamma \in \{SS₁, SS₂, SRₓ\}, \quad z \in \{s₁, s₂\}, \quad n \in \{1, 2, ..., N\} \).

The selected secondary relay SRₓ applies the SIC procedure which was assigned in the set up phase to decode successively the data from \( s₁ \) to \( s₂ \), the SS₁ cannot get the data \( s₂ \) in the three cases as: 1) the SRₓ cannot decode the first-order data \( s₁ \) (denoted as \( R_{SRₓ\rightarrow s₁} < R_{th} \)), 2) the SRₓ gets the data \( s₁ \) successfully but does not decode the second-order data \( s₂ \) (denoted as \( R_{SRₓ\rightarrow s₁} ≥ R_{th} \) \( R_{SRₓ\rightarrow s₂} < R_{th} \)), or 3) the SRₓ gets both \( s₁ \) and \( s₂ \) successfully but the SS₁ cannot decode the desired data \( s₂ \) in the second time slot (denoted as \( R_{SRₓ\rightarrow s₁} ≥ R_{th} \) \( R_{SRₓ\rightarrow s₂} ≥ R_{th} \) \( R_{SS₁\rightarrow s₂} < R_{th} \)).

By summing the above cases, the outage probability of the secondary source SS₁ is expressed mathematically as

\[
OP_{SS₁} = \Pr \left[ R_{SRₓ\rightarrow s₁} < R_{th} \right] + \Pr \left[ \left( R_{SRₓ\rightarrow s₁} ≥ R_{th} \right) \cap \left( R_{SRₓ\rightarrow s₂} < R_{th} \right) \right] + \Pr \left[ \left( R_{SRₓ\rightarrow s₁} ≥ R_{th} \right) \cap \left( R_{SS₁\rightarrow s₂} < R_{th} \right) \right] \quad (9)
\]
where \( \text{Pr}[\Xi] \) mean probability operations of events \( \Xi \).

**Lemma 1:** The probability \( \Delta_1 \) is solved as

\[
\Delta_1 = 1 + \frac{\Psi_5 \sum_{p=1}^{N} \left( \frac{p}{N} \right) (-1)^p \Psi_4 \nu_4(p)}{\Psi_5 \nu_3(p) - \Psi_4 \nu_4(p)} \times \left( 1 - \frac{\Psi_5 \nu_4(p)}{\Psi_5 \nu_3(p) - \Psi_4 \nu_4(p)} \ln \left( \frac{\Psi_3 \nu_3(p)}{\Psi_2 \nu_4(p)} \right) \right),
\]

and where:

\[
\nu_1 = (2^R + 1) \frac{\psi_4}{\psi_5}; \quad \nu_2 = (2^R - 1)(\psi_4 + \psi_5)/(\psi_5 \psi_6) \quad \text{functions}
\]

\( \nu_3(p) \) and \( \nu_4(p) \) are defined respectively as

\[

\nu_3(p) = (\psi_1 + p \psi_4 \nu_2)/\psi_1 \psi_4
\]

\[

\nu_4(p) = \frac{p}{N} \psi_1
\]

denotes the binomial coefficient \( \binom{p}{N} = \frac{N!}{p!(N-p)!} \).

**Proof:** Proven in Appendix A.

**Lemma 2:** The probability \( \Delta_2 \) is given in two cases of the perfect (\( \varepsilon = 0 \)) and imperfect SIC (\( \varepsilon = 1 \)) operations as formula at the top of next page.

In the formulas (11), \( \Gamma(a,b) \) is an incomplete Gamma function [38] (see the equation (8.350.2)), \( \nu_5 = (2^R - 1)(\psi_4 + \psi_5)/(\psi_5 \psi_6) \),

\[
\nu_6 = (\psi_2 + \psi_5)/(\psi_5 \psi_6)
\]

and a function

\[

\nu_7(p) = (\nu_2(p) + \nu_3(p))/\psi_1
\]

Proof of the Lemma 2 is provided in Appendix B.

The event \( R_{S_{\text{th}} < R_h} \) happens independently with the intersection event of \( R_{S_{\text{th}} \geq R_h} \) and \( R_{S_{\text{th}} > R_h} \), thus we have an equivalent representation of the probability \( \Delta_3 \) in (9) as formulas (12).

\[
\Delta_3 = \text{Pr}\left[R_{S_{\text{th}}<R_h} \cap \left[R_{S_{\text{th}}<R_h} \cap R_{S_{\text{th}}=R_h}\right]\right]
\]

\[
= (1 - \Delta_1 - \Delta_3) \times \text{Pr}\left[R_{S_{\text{th}}<R_h}\right]
\]

The probability \( \text{Pr}\left[R_{S_{\text{th}}<R_h}\right] \) is solved with referring the formula (A.2) as

\[
\text{Pr}\left[R_{S_{\text{th}}<R_h}\right] = \text{Pr}\left[\frac{1}{2} \log_2 \left(1 + \frac{Q_{S_{\text{th}}}}{g_{S_{\text{th}}}} \right) < R_h\right]
\]

\[
= \text{Pr}\left[\frac{Q_{S_{\text{th}}} \psi_2}{g_{S_{\text{th}}} (\psi_1 + \psi_2)} < 2^R - 1\right] = \text{Pr}\left[\frac{g_{S_{\text{th}}} (\psi_1 + \psi_2)}{Q_{S_{\text{th}}} \psi_2} < 2^R - 1\right]
\]

\[
= \left(\psi_1 + \psi_2\right) \psi_3 (2^R - 1)
\]

\[
\text{Pr}\left[R_{S_{\text{th}}<R_h}\right] = \text{Pr}\left[\frac{1}{2} \log_2 \left(1 + \frac{Q_{S_{\text{th}}}}{\psi_2 + \psi_2}\right) < R_h\right]
\]

\[
= \text{Pr}\left[\frac{Q_{S_{\text{th}}} \psi_2}{g_{S_{\text{th}}} (\psi_1 + \psi_2)} < 2^R - 1\right] = \text{Pr}\left[\frac{g_{S_{\text{th}}} (\psi_1 + \psi_2)}{Q_{S_{\text{th}}} \psi_2} < 2^R - 1\right]
\]

\[
= \left(\psi_1 + \psi_2\right) \psi_3 (2^R - 1)
\]
Substituting (12) into (9) and using (13), the outage probability of the secondary source SS1 is analyzed by a closed-form expression as

$$\begin{align*}
OP_{SS1} &= \frac{\left(\Psi_1 + \Psi_2\right)\Psi_3\left(2^{2^\Delta_1} - 1\right) + \left(\Delta_1 + \Delta_2\right)\Psi_1\Psi_2 Q}{\Psi_1\Psi_2 Q + \left(\Psi_1 + \Psi_2\right)\Psi_3\left(2^{2^\Delta_3} - 1\right)}
\end{align*}$$

where \(\Delta_1\) and \(\Delta_2\) are provided by the Lemmas 1 and 2, respectively.

The SS2 cannot get the desired data \(s_j\) in only the two cases as: 1) the SRs cannot also decode the first-ordered data \(s_j\), 2) the SRs transmits the signals containing the decoded data \(s_j\) but the SS2 cannot get the \(s_j\) (denoted as \(R_{SR_j \rightarrow SS_j} \geq R_{th}\) \(\cap R_{SS_j \rightarrow SS_j} < R_{th}\)). Therefore, the outage probability of the secondary source SS2 is presented and solved as

$$\begin{align*}
OP_{SS2} &= \Pr[R_{SR_j \rightarrow SS_j} < R_{th}] \\
&\quad + \Pr[R_{SR_j \rightarrow SS_j} \geq R_{th}] \Pr[R_{SS_j \rightarrow SS_j} < R_{th}] \\
&= \Delta_1 + \Pr[R_{SR_j \rightarrow SS_j} \geq R_{th}] \Pr[R_{SS_j \rightarrow SS_j} < R_{th}] \\
&= \Delta_1 + \left(1 - \Delta_1\right) \frac{\left(\Psi_1 + \Psi_2\right)\Psi_3\left(2^{2^\Delta_3} - 1\right) + \left(\Delta_1 + \Delta_2\right)\Psi_1\Psi_2 Q\Delta_1}{\Psi_1\Psi_2 Q + \left(\Psi_1 + \Psi_2\right)\Psi_3\left(2^{2^\Delta_3} - 1\right)}
\end{align*}$$

where \(\Delta_1\) is provided by the Lemma 1.

From (13-14), we notice that by allocating the transmit power ratios for the data \(s_1\) and \(s_2\) in the SC-coded signal \(s\), the decoding outage probabilities at the secondary sources SS1 and SS2 from the selected secondary relay SRs are identical, i.e. \(\Pr[R_{SS_j \rightarrow SS_j} < R_{th}] = \Pr[R_{SS_j \rightarrow SS_j} < R_{th}]\).

In addition, from (14-15), we have a result as \(OP_{SS1} \geq OP_{SS2}\) due to additional data decoding of \(s_j\) at the selected secondary relay in the \(OP_{SS1}\).

Finally, the outage probabilities of the SS1 and SS2 in the RRS protocol (denoted as \(OP_{SS1}^{RRS}\) and \(OP_{SS2}^{RRS}\)) are expressed and solved in the similar approach as in the proposed SIC-SC protocol. Hence, these outage probabilities are obtained as

$$\begin{align*}
OP_{SS1}^{RRS} &= \Pr[R_{SR_j \rightarrow SS_j} < R_{th}] \\
&\quad + \Pr[R_{SR_j \rightarrow SS_j} \geq R_{th}] \Pr[R_{SS_j \rightarrow SS_j} < R_{th}] \\
&= \Delta_4 + \Pr[R_{SR_j \rightarrow SS_j} \geq R_{th}] \Pr[R_{SS_j \rightarrow SS_j} < R_{th}] \\
&= \frac{\left(2^{2^\Delta_3} - 1\right)\Psi_3 + \phi_3 \Psi_2 Q\Delta_4}{\left(2^{2^\Delta_3} - 1\right)\Psi_3 + \phi_3 \Psi_2 Q}
\end{align*}$$

where \(\Delta_4\) and \(\Delta_5\) are inferred from Lemmas 1 and 2, and are presented by the below formula (18) and the formula (19) at the top of this page.

\(\Delta_4 = 1 - \frac{\Psi_5}{\Psi_4 \left(\tau_3 \Psi_5 - \tau_4 \Psi_2\right)} \times \left(1 - \frac{\tau_4 \Psi_2}{\tau_3 \Psi_5 - \tau_4 \Psi_2} \ln\left(\frac{\tau_3 \Psi_5}{\tau_4 \Psi_2}\right)\right)\) (18)

In the formulas (18-19), \(\tau_i = \left(2^{2^\Delta_3} - 1\right)\phi_i / \phi_1\), \(\tau_2 = \tau_1 / \left(\phi_2 Q\right)\), \(\tau_3 = \left(\Psi_1 + \tau_2 \Psi_4\right) / \left(\Psi_1 \Psi_4\right)\), \(\tau_4 = \tau_1 / \Psi_1\), \(\tau_5 = \tau_5 \phi_1 / \phi_2\), \(\tau_6 = \left(\Psi_1 + \tau_2 \Psi_4\right) / \left(\tau_3 \Psi_5 \Psi_2 Q\right)\) and \(\tau_7 = \left(\tau_3 + \tau_4 \tau_5\right) / \left(\tau_4 \tau_5 \Psi_6 Q\right)\).
\[\Delta_s = \begin{cases} \frac{\Psi_5}{\Psi_4(\tau_3 \Psi_5 - \tau_4 \Psi_2)} & \times \left( \frac{\tau_5 \Psi_5 - \frac{\tau_4 \Psi_2}{\tau_3 \Psi_5 - \tau_4 \Psi_2}}{\ln \left( \frac{\tau_3(\Psi_2 + \tau_5 \Psi_5)}{(\tau_3 + \tau_4 \tau_5) \Psi_2} \right) } \right), & \varepsilon = 0 \\
\frac{e^{\tau_0} \Gamma[0, \tau_0]}{\Psi_5 \Psi_6} & \times \frac{1}{\tau_5 \Psi_5 \Psi_6^{-1}} \times \frac{\tau_3 \Psi_5 - \tau_4 \Psi_2}{\tau_6} \times \left( \frac{\tau_3 \Psi_5 - \tau_4 \Psi_2}{\tau_6} \right)^{2} \times \left( \frac{\tau_3 \Psi_5 - \tau_4 \Psi_2}{\tau_6} \right)^{-2} \times \left( \frac{\tau_3 \Psi_5 - \tau_4 \Psi_2}{\tau_6} \right)^{-2} \times \left( \frac{\tau_3 \Psi_5 - \tau_4 \Psi_2}{\tau_6} \right)^{-2} \times \left( \frac{\tau_3 \Psi_5 - \tau_4 \Psi_2}{\tau_6} \right)^{-2}, & \varepsilon = 1 \end{cases} \]

4. Results and Discussions

This section presents analysis and simulation results of the proposed SIC-SC protocol and the RRC comparison protocol with two cases of perfect SICs (\(\varepsilon = 0\)) and imperfect SICs (\(\varepsilon = 1\)). The Monte Carlo simulation results are made to verify the theoretical derivations in the section 3. The values for \(\Psi_{m}, m \in \{1, 2, 3, 4, 5, 6\}\) are referenced in [12, 18, 29, 39, 40]. In all the subsequent results, the threshold data rate \(R_{th}\) is fixed to 3 (bps/Hz) and the normalized variance of the additive white Gaussian noises \((N_0)\) is set to 1. In addition, blue and red markers denote simulated values of the secondary sources SS1 and SS2, respectively, and black solid lines present theoretical analyses.

Figure 2 presents the outage probabilities of the secondary sources SS1 and SS2 in the protocols SIC-SC and RRS versus \(Q\) (dB) when \(\Psi_1 = 20\) (dB), \(\Psi_2 = 10\) (dB), \(\Psi_3 = 1\) (dB), \(\Psi_4 = 1\) (dB), \(\Psi_5 = 0.5\) (dB), \(\Psi_6 = -5\) (dB) and number of the secondary relays is examined at 5 and 8 \((N \in \{5, 8\})\). Power allocation parameters for the RRS comparison protocol are set to fixed values as \(\phi_1 = 0.3, \phi_2 = 1, \phi_3 = 0.7, \phi_4 = 0.4\) and \(\phi_5 = 1 - \phi_3 = 0.6\). As shown in the Figure 2, some observations for the proposed SIC-SC protocol are listed as follows. Firstly, the outage probabilities of the secondary source SS1 and SS2 decrease when the number of the secondary relays \(N\) and interference constraint \(Q\) increase in two cases of the perfect SICs and imperfect SICs. Secondly, these probabilities move to saturation values at the high \(Q\) regions, e.g. \(Q > 30\) (dB) when \(N = 8\). Thirdly, the outage performance of the secondary source SS2 is better than that of the secondary source SS1. Fourthly, the outage probability of the secondary source SS2 is not affected by the cases of the SIC operations and the secondary source SS1 achieves smaller outage probabilities in the perfect SICs. Fifthly, the proposed SIC-SC protocol outperforms the RRS comparison protocol where the random relay selection and fixed power allocations are considered. Finally, the derived theory values (black solid lines) are precise to the simulation ones (marker symbols). These conclusions are explained by increasing the diversity amount and the transmit powers. In addition, the received SINRs at the secondary source SS1 are weakened by combining effects of the interference component from the data-carried signal, the residual interference by imperfect SICs and interference constraints of the primary network whereas the perfect and imperfect SIC events existed in the received SINR \(\gamma_{SR_{s} \rightarrow S}\) as in (5) and (8) are not considered in taking the own data \(s_2\) of the secondary source SS2 (see the formula (15)). One more thing, the RRS protocol does not depend on the number of the secondary relays and can be used if there is at least one cooperative secondary relay.
Figure 2. Outage probabilities of the SIC-SC and RRS protocols versus $Q$ (dB) when $\Psi_1 = 20$ (dB), $\Psi_2 = 10$ (dB), $\Psi_3 = 1$ (dB), $\Psi_4 = 1$ (dB), $\Psi_5 = 0.5$ (dB), $\Psi_6 = -5$ (dB), $\phi_1 = 0.3$, $\phi_2 = 0.7$, $\phi_3 = 0.4$, $\phi_4 = 0.6$ and $N \in \{5, 8\}$.

Figure 3 presents the outage probabilities versus $\Psi_4$ (dB) when $\Psi_1 = 20$ (dB), $\Psi_2 = 10$ (dB), $\Psi_3 = 1$ (dB), $\Psi_5 = 0.5$ (dB), $\Psi_6 = -5$ (dB), $\phi_1 = 0.3$, $\phi_2 = 0.7$, $\phi_3 = 0.4$, $\phi_4 = 0.6$, $N = 8$ and $Q = 10$ (dB).

As shown in Figure 4, the system performance of the protocols SIC-SC and RRS is enhanced according the increasing of the channel power $\Psi_1$ because of high-success decoding of the first data in the SIC technology. In addition, the lower residual interference parameters lead to the higher performances of the secondary source SS$_1$. The case of the perfect SICs is viewed as $\Psi_6 \to -\infty$ (dB). In both protocols SIC-SC and RRS, the SIC and SC technologies improve the spectrum utilization efficiency by decreasing the number of the time slots from the secondary sources to the secondary relays and vice versa, but the SIC-SC protocol performs better by the adaptive power allocation parameters $\Psi_1/(\Psi_1 + \Psi_2)$ and $\Psi_2/(\Psi_1 + \Psi_2)$ in the coded $s$ as a function of the channel powers $\Psi_i$. 

Figure 4 shows the outage probabilities of the SIC-SC and RRS protocols versus $\Psi_4$ (dB) in situations as $\Psi_2 = 10$ (dB), $\Psi_3 = 1$ (dB), $\Psi_5 = 0.5$ (dB), $\Psi_6 = -5$ (dB), $\phi_1 = 0.3$, $\phi_2 = 0.7$, $\phi_3 = 0.4$, $\phi_4 = 0.6$, $N = 8$ and $Q = 10$ (dB).
Figure 4. Outage probabilities of the SIC-SC and RRS protocols versus $\Psi_1$ (dB) when $\Psi_2 = 10$ (dB), $\Psi_3 = 1$ (dB), $\Psi_4 = 0.5$ (dB), $\phi_1 = 0.3$, $\phi_2 = 0.7$, $\phi_3 = 0.4$, $\phi_4 = 0.6$, $N = 8$, $Q = 10$ (dB) and $\Psi_s \in \{-10\,\text{dB}, -5\,\text{dB}\}$.

5. Conclusions

In this paper, we analyzed the underlay two-way DF scheme in which the secondary relays use the SIC technology to decode the data of two secondary sources sequentially, and then make a coded signal by the SC technology, known as the SIC-SC protocol. The SIC-SC protocol was designed to operate in two time slots under effects from the interference constraint of the primary receiver and residual interference of the imperfect SIC processes. Transmit powers provided to carry the data were allocated dynamically according to channel powers of interference and transmission. The secondary relay was selected from considering strongest channel gain subject to increase decoding capacity of the first data and decrease collection time of the CSIs. The identical underlay two-way DF operation with the random relay selection and the fixed power allocations (called the RRS protocol) was also investigated to compare the proposed SIC-SC protocol. The closed-form outage probability expressions were derived from mathematical manipulations and verified exactly by performing the Monte Carlo simulations. The simulation and analysis results shown that 1) the system performance of the proposed SIC-SC protocol in terms of the outage probabilities was enhanced by increasing the number of the secondary relays and the interference constraint as well as decreasing the residual interference powers, and 2) the non-identical outage performances of the secondary sources depended on both interference channel powers to the primary receiver and desired channel powers to the selected secondary relay from the secondary sources, and 3) the performance of the nearer secondary source is worse than that of the distant secondary source, and finally, the system performance of the proposed SIC-SC protocol outperforms that of the RRS protocol in terms of the outage probabilities.

Appendix A: Proof of Lemma 1

Substituting (7) into the probability of $\Delta_1$ as in (9), the $\Delta_1$ is expressed as

$$\Delta_1 = \Pr \left[ \frac{1}{2} \log_2 \left( 1 + \frac{R_{x_{SR}}}{{\Psi}_1} \right) < R_{th} \right]$$

$$= \Pr \left[ \frac{R_{x_{SR}}}{{\Psi}_1} < 2^{2R_{th} - 1} \right]$$

$$= \Pr \left[ \frac{R_{x_{SR}}}{{\Psi}_1} < 2^{2R_{th} - 1} \frac{Q_{s_{SR}}}{{\Psi}_1} \right]$$

$$= \Pr \left[ \frac{R_{x_{SR}}}{{\Psi}_1} < 2^{2R_{th} - 1} \frac{Q_{s_{SR}}}{{\Psi}_1} \frac{\Psi_5}{\Psi_1} \right]$$

$$= \Pr \left[ \frac{R_{x_{SR}}}{{\Psi}_1} < 2^{2R_{th} - 1} \frac{Q_{s_{SR}}}{{\Psi}_1} \frac{\Psi_5}{\Psi_1} \frac{\Psi_4}{\Psi_1} \right]$$

$$= \Pr \left[ \frac{R_{x_{SR}}}{{\Psi}_1} < 2^{2R_{th} - 1} \frac{Q_{s_{SR}}}{{\Psi}_1} \frac{\Psi_5}{\Psi_1} \frac{\Psi_4}{\Psi_1} \frac{\Psi_4}{\Psi_1} \right]$$

$$= \Pr \left[ \frac{R_{x_{SR}}}{{\Psi}_1} < 2^{2R_{th} - 1} \frac{Q_{s_{SR}}}{{\Psi}_1} \frac{\Psi_5}{\Psi_1} \frac{\Psi_4}{\Psi_1} \frac{\Psi_4}{\Psi_1} \right]$$

$$= \int_0^{\frac{R_{x_{SR}}}{{\Psi}_1}} f_{x_{SR}/g_{SR}}(x) f_{x_{SR}/g_{SR}}(\frac{\Psi_5}{\Psi_1}) \frac{\Psi_4}{\Psi_1} \frac{\Psi_4}{\Psi_1} dx,$$

where $f_{x_{SR}/g_{SR}}(x)$ are $F_{x_{SR}/g_{SR}}(x)$ are the PDF and CDF of the RVs $g_x/g_y$, $X,Y \in \{s_1,s_2,s_{SR},pr\}$, $n \in \{1,2,...,N\}$.

By referring from [29] (see equations (24–25)), the $f_{x_{SR}/g_{SR}}(x)$ is obtained as
\[ f_{\xi_{SS,SR}/\xi_{SS,PR}}(x) = \frac{\partial f_{\xi_{SS,SR}/\xi_{SS,PR}}}{\partial x}(x) \]
\[ = \frac{\Psi_{2}\Psi_{s}}{(\Psi_{2} + \Psi_{s}x)^{2}}. \]  
(A.2)

The \( F_{\xi_{SS,SR}/\xi_{SS,PR}}(x) \) is expressed as
\[ F_{\xi_{SS,SR}/\xi_{SS,PR}}(x) = \text{Pr}\left[ \xi_{SS,SR} < X \right] \]
\[ = \text{Pr}\left[ \xi_{SS,SR} < x \xi_{SS,PR} \right] \]
\[ = \int_{0}^{x} f_{\xi_{SS,PR}}(y) F_{\xi_{SS,SR}}(xy) dy, \]  
where \( F_{\xi_{SS,SR}}(x) \) is the CDF of the RV \( \xi_{SS,SR} \) and is expressed as (see the equation (7–14) in [24])
\[ F_{\xi_{SS,SR}}(x) = (1-e^{-pN}) \times \sum_{p=0}^{N} \left( \frac{p}{N} \right)(-1)^{p} e^{-pN} \]  
(A.4)

In (A.4), \( \left( \frac{p}{N} \right) \) denotes the binomial coefficient \( \left( \frac{N!}{p!(N-p)!} \right) \).

Then, the \( F_{\xi_{SS,SR}/\xi_{SS,PR}}(x) \) is solved as
\[ F_{\xi_{SS,SR}/\xi_{SS,PR}}(x) = e^{-pN} \times \sum_{p=0}^{N} \left( \frac{p}{N} \right)(-1)^{p} e^{-pN} dy \]  
\[ = \Psi_{2} \times \sum_{p=0}^{N} \left( \frac{p}{N} \right)(-1)^{p} . \]  
(A.5)

Substituting (A.2) and (A.5) into (A.1), the \( \Delta_{1} \) is manipulated equivalently as
\[ \Delta_{1} = \int_{0}^{x} \Psi_{2} \Psi_{s} \times \sum_{p=0}^{N} \left( \frac{p}{N} \right)(-1)^{p} \delta dx \]
\[ = \Psi_{2} \Psi_{s} \times \sum_{p=0}^{N} \left( \frac{p}{N} \right)(-1)^{p} \delta dx \]  
(A.6)

By performing variable transformations as \( y = (\Psi_{2} + \Psi_{s}x)^{-1} \) and \( x = u_{1}(p) + \frac{\left( \Psi_{2}u_{2}(p) - \Psi_{s}u_{1}(p) \right)}{\Psi_{2} \Psi_{s}} \)

, where \( u_{1}(p) = \left( \Psi_{2} + p \Psi_{2}u_{2} \right) / \left( \Psi_{2} \Psi_{s} \right) \) and \( u_{2}(p) = pu_{1}(p) \), the Lemma 1 is proven completely.

**Appendix B: Proof of Lemma 2**

From (8) and (A.1), the probability \( \Delta_{2} \) as in (9) is expressed as
\[ \Delta_{2} = \text{Pr}\left( \left[ \frac{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}}{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}} \right] \geq 2^{\lambda_{0}} - 1 \right) \]
\[ \cap \left[ \frac{1}{2} \log_{2} \left( 1 + \gamma_{SS,PR} \right) < R_{0} \right] \]
\[ = \text{Pr}\left( \left[ \frac{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}}{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}} \right] \geq 2^{\lambda_{0}} - 1 \right) \]
\[ \cap \left[ \frac{1}{2} \log_{2} \left( 1 + \gamma_{SS,PR} \right) < R_{0} \right] \]
\[ = \text{Pr}\left( \frac{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}}{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}} \right) \]
\[ \cap \left[ \frac{1}{2} \log_{2} \left( 1 + \gamma_{SS,PR} \right) < R_{0} \right] \]
\[ = \text{Pr}\left[ \frac{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}}{\Psi_{2}Q_{\xi_{SS,SR}/\xi_{SS,PR}} + (\Psi_{2} + \Psi_{s})_{\xi_{SS,SR}/\xi_{SS,PR}}} \right] \]
\[ \cap \left[ \frac{1}{2} \log_{2} \left( 1 + \gamma_{SS,PR} \right) < R_{0} \right] \]  
(B.1)

To solve the \( \Delta_{2} \) in (B.1) by closed-form expressions, we consider two cases of perfect SICs \( (\varepsilon = 0) \) and imperfect SICs \( (\varepsilon = 1) \) as follows.

- **Perfect SICs \( (\varepsilon = 0) \):** By referring from (A.2) and using (A.5), the \( \Delta_{2} \) is obtained as
\[ \Delta_{2} = \int_{0}^{x} \left( 1 - F_{\xi_{SS,SR}/\xi_{SS,PR}}(u_{1}(x + u_{2})) \right) dx \]
\[ = \int_{0}^{x} \Psi_{2}u_{2} \Psi_{s} \sum_{p=0}^{N} \left( \frac{p}{N} \right)(-1)^{p} \delta dx \]  
(B.2)

Also performing as (A.6), \( \Delta_{2} \) is solved as in Lemma 2 with the case \( \varepsilon = 0 \).

- **Imperfect SICs \( (\varepsilon = 1) \):** The \( \Delta_{2} \) in this case \( (\varepsilon = 1) \) is presented as
\[
\Delta_2 = \int_0^\infty f_{\xi_2}(x) \int_0^\infty \left(1 - F_{\xi_{1,2k},\xi_{1,2m}}(\xi_1 y + \xi_2)\right) \times f_{\xi_{1,2k},\xi_{1,2m}}(y) dy dx \\
= \int_0^\infty \rho^{-1/\nu} \left(\frac{\psi_{\xi_1}(1 + \rho\xi)}{\psi_{\xi_2} + \psi_{\xi_3}(1 + \rho\xi)}\right)^{-1} \times \sum_{p \neq \xi_1} \frac{\psi_{\xi_2}(p) - \psi_{\xi_3}(p)}{\psi_{\xi_2}(1 + \rho\xi) - \psi_{\xi_3}(1 + \rho\xi)} \times \ln\left(\frac{\psi_{\xi_2}(\xi_1 p) + \psi_{\xi_3}(\xi_1 p)}{\psi_{\xi_2}(1 + \rho\xi) + \psi_{\xi_3}(1 + \rho\xi)}\right) \times f_{\xi_2}(x) dx.
\]

By extending \(\ln(\cdot)\), changing variables and then solving the integrals in (B.3), the probability \(\Delta_2\) is answered as in Lemma 2 with the remaining case \(\varepsilon = 1\). Hence, the Lemma 2 is verified completely.

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