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Max - Min Composition of Linguistic Intuitionistic Fuzzy Relations and Application in Medical Diagnosis[1]

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Abstract

In this paper, we first introduce the notion of linguistic intuitionistic fuzzy relation. This notion is useful in situations when each correspondence of objects is presented as two labels such that the first expresses the degree of membership, and the second expresses the degree of non-membership as in the intuitionistic fuzzy theory**.** Sanchez's approach for medical diagnosis is extended using the linguistic intuitionistic fuzzy relation.

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1. Introduction

The correspondences between objects can be suitably described as relations. A traditional crisp relation represents the satisfaction or the dissatisfaction of relationship, connection or correspondence between the objects of two or more sets. This concept can be extended to allow for various degrees or strengths of relationship or connection between objects. Degrees of relationship can be represented by membership grades in a fuzzy relation [1] in the same way as degrees of membership are represented in the fuzzy set [2]. However, there is a hesitancy or a doubtfulness about the grades assigned to the relationships between objects. In fuzzy set theory, there is no mean to deal with that hesitancy in the membership grades. A reasonable approach is to use intuitionistic fuzzy sets defined by Atanassov in 1983 [3-4]. Motivated by intuitionistic fuzzy sets theory, in 1995 [5], Burillo and Bustince first proposed intuitionistic fuzzy relation. Further researches of this type of relation can be found in [6-9].

There are many situations, due to the natural aspect of the information, the information cannot be given precisely in a quantitative form but in a qualitative one [10]. Thus, in such situations, a more realistic approach is to use linguistic assessments instead of numerical values by mean of linguistic labels which are not numbers but words or sentences in a natural or artificial language [11].

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One of the main concepts in relational calculus is the composition of relations. This makes a new relation using two relations. For example, relation between patients and illnesses can be obtained from relation between patients and symptoms and relation between symptoms and illnesses (see medical diagnosis [8, 12-13]).

In this paper, we define linguistic intuitionistic fuzzy relation which is an extension of intuitionistic fuzzy relation using linguistic labels. Then, we propose $\max - \min$ composition of the linguistic intuitionistic fuzzy relations. Finally, an application in medical diagnosis is introduced.

2. Preliminaries

In this section, we give some basic definitions used in next sections.

2.1. Intuitionistic fuzzy set

Intuitionistic fuzzy set, a significant generalization of fuzzy set, can be useful in situations when description of a problem by a linguistic variable, given in terms of a membership function only, seems too rough. For example, in decision making problems, particularly in medical diagnosis, sales analysis, new product marketing, financial services, etc., there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object.

Definition 2.1. [3] An intuitionistic fuzzy set A on a universe X is an object of the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\},$$

where $\mu_A(x) \in [0,1]$ is called the "degree of membership of x in A", $\nu_A(x) \in [0,1]$ is called the "degree of non-membership of x in A", and the following condition is satisfied

$$\mu_A(x) + \nu_A(x) \leq 1 \ \forall x \in X.$$

Some developments of the intuitionistic fuzzy sets theory with applications, for examples, can be seen in [5, 8, 14-18].

2.2. Linguistic Labels

In many real world problems, the information associated with an outcome and state of nature is at best expressed in term of linguistic labels [19-21]. One of the approaches is to let experts give their opinions using linguistic labels. In order to deploy the above approach, they have been using a finite and totally ordered discrete linguistic label set $S = \{s_1, s_2, \ldots, s_n\}$. Where n is an odd positive integer, s_i represents a possible value for a linguistic variable, and it requires that [21]:

- The set is ordered: $s_i \geq s_j$ iff $i \geq j$;

- The negation operator is defined as: $neg(s_i) = s_j$ such that $j = n + 1 - i$.

For example, a set of seven linguistic labels S could be defined as follows [10]:

$$S = \{s_1 = none, s_2 = very\ low, s_3 = low, s_4 = medium, s_5 = high, s_6 = very\ high, s_7 = perfect\}.$$

An overview of linguistic aggregation operators which handle linguistic labels is given in [22].

2.3. Intuitionistic fuzzy relations

1) Intuitionistic fuzzy relations

Intuitionistic fuzzy relation, an extension of fuzzy relation, was first introduced by Burillo and Bustince in 1995.

Definition 2.2. [5] Let X, Y be ordinary finite non-empty sets, an intuitionistic fuzzy relation (IFR) R between X and Y is defined as an intuitionistic fuzzy set on $X \times Y$, that is, R is given by:

$$R=\left\{\langle (x,y),\mu_R(x,y),\nu_R(x,y)\rangle \middle| (x,y)\in X\times Y\right\},$$

where μ_R, $\nu_R : X\times Y\to[0,1]$ satisfy the condition

$$\mu_R(x,y)+\nu_R(x,y)\le 1, \forall (x,y)\in X\times Y.$$

The set of all IFR between X and Y is denoted by $IFR(X\times Y)$.

2) Composition of intuitionistic fuzzy relations

Triangular norm and triangular co-norm are notions used in the framework of probabilistic metric spaces and in multi-valued logic, specifically in fuzzy logic.

Definition 2.3. 1. A triangular norm (t-norm) is a commutative, associative, increasing $[0,1]^2\to[0,1]$ mapping T satisfying $T(x,1)=x$, for all $x\in[0,1]$.

2. A triangular conorm (t-conorm) is a commutative, associative, increasing $[0,1]^2\to[0,1]$ mapping S satisfying $S(0,x)=x$, for all $x\in[0,1]$.

In 1995 [5], Burillo and Bustince introduced concepts of intuitionistic fuzzy relation and compositions of intuitionistic fuzzy relations using four triangular norms or co-norms.

Definition 2.4. [5] Let α, β, λ, ρ be four t-norms or t-conorm, $R\in IFR(X\times Y)$, $P\in IFR(Y\times Z)$. Relation $P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R\in IFR(X\times Z)$ is defined as follows:

$$P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R=\left\{\left\langle (x,z),\mu_{P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R}(x,z),\nu_{P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R}(x,z)\right\rangle \middle| (x,z)\in X\times Z\right\},$$

where

$$\mu_{P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R}(x,z)=\underset{y}{\alpha}\left\{\beta\left[\mu_R(x,y),\mu_P(y,z)\right]\right\},$$

$$\nu_{P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R}(x,z)=\underset{y}{\lambda}\left\{\rho\left[\nu_R(x,y),\nu_P(y,z)\right]\right\},$$

whenever

$$\mu_{P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R}(x,z)+\nu_{P\underset{\lambda,\rho}{\overset{\alpha,\beta}{\circ}}R}(x,z)\le 1, \forall (x,z)\in X\times Z.$$

Consider the set L^* and the operation $\le_{L^*}$ defined by:

$$L^*=\left\{(x_1,x_2) \middle| (x_1,x_2)\in[0,1]^2 \text{ and } x_1+x_2\le 1\right\},$$

$$(x_1,x_2)\le_{L^*}(y_1,y_2)\Leftrightarrow x_1\le y_1 \text{ and } x_2\ge y_2,$$

$$\forall (x_1,x_2),\ (y_1,y_2)\in L^*.$$

Then, $\langle L^*,\le_{L^*}\rangle$ is a complete lattice [23]. Using the relation $\le_{L^*}$, the minimum and the maximum are defined. They are denoted by $0_{L^*}=(0,1)$ and $1_{L^*}=(1,0)$, respectively. In the followings, intuitionistic fuzzy triangular norm and intuitionistic triangular conorm, an extension of fuzzy relation, are recalled.

Definition 2.5. [24] 1. An intuitionistic fuzzy triangular norm (it-norm) is a commutative, associative, increasing $L^{*2}\to L^*$ mapping T satisfying $\mathrm{T}(x,1_{L^*})=x$ for all $x\in L^*$.

2. An intuitionistic fuzzy triangular conorm (it-conorm) is a commutative, associative, increasing $L^{*2}\to L^*$ mapping S satisfying $\mathrm{S}(x,0_{L^*})=x$ for all $x\in L^*$.

In [7], we defined a new composition of intuitionistic fuzzy relations using two it-norms or it-conorms. Using the new composition, if we make a change in non-membership components of two relations, the membership components of the result may change, which is more realistic. We also proved the Burillo and Bustince's notion is a special case of our notion, stated many properties.

3. Linguistic Intuitionistic Fuzzy Relations

A. Linguistic Intuitionistic Labels

In decision making problems, particularly in medical diagnosis, sales analysis, new product marketing, financial services, etc., there is a hesitation part at each moment of the evaluation of an object. In this case, the information can be expressed in terms of pair of labels, where one label represents the degree of membership and the second represents the degree of non-membership.

For example, in medical diagnosis, an expert can assess the correspondence between patient p and symptom q as a pair (s_i, s_j), where $s_i \in S$ is the degree of membership of the patient p in the set of all patients suffered from the symptom q, and $s_j \in S$ is the degree of non-membership of the patient p in this set.

In [16], we first proposed the notion of intuitionistic label to present experts' assessments in these situations.

Definition 3.1. [16] A linguistic intuitionistic label is defined as a pair of linguistic labels $(s_i, s_j) \in S^2$ such that $i + j \le n+1$, where $S = \{s_1, s_2, \ldots, s_n\}$ is the linguistic label set, s_i, $s_j \in S$ respectively define the degree of membership and the degree of non-membership of an object in a set.

The set of all linguistic intuitionistic labels is denoted as IS, i.e.

$$IS = \left\{(s_i, s_j) \in S^2 \,\middle|\, i + j \le n+1\right\}.$$

For example, if the linguistic label set S contains $s_1 = none$, $s_2 = very\ low$, $s_3 = low$, $s_4 = medium$, $s_5 = high$, $s_6 = very\ high$, and $s_7 = perfect$, the corresponding linguistic intuitionistic label set IS is given as in table I.

In [16], we also defined some lexical order relations on IS: the membership-based order relation and the non-membership-based order relation.

TABLE I
LINGUISTIC INTUITIONISTIC LABEL SET

(s_7,s_1)						
(s_6,s_1)	(s_6,s_2)					
(s_5,s_1)	(s_5,s_2)	(s_5,s_3)				
(s_4,s_1)	(s_4,s_2)	(s_4,s_3)	(s_4,s_4)			
(s_3,s_1)	(s_3,s_2)	(s_3,s_3)	(s_3,s_4)	(s_3,s_5)		
(s_2,s_1)	(s_2,s_2)	(s_2,s_3)	(s_2,s_4)	(s_2,s_5)	(s_2,s_6)	
(s_1,s_1)	(s_1,s_2)	(s_1,s_3)	(s_1,s_4)	(s_1,s_5)	(s_1,s_6)	(s_1,s_7)

Definition 3.2. [16] For all (μ_1,ν_1), (μ_2,ν_2) in IS, membership-based order relation $\ge_M$ and non-membership- based order relation $\ge_N$ are defined as following

$$(\mu_1,\nu_1) \ge_M (\mu_2,\nu_2) \Leftrightarrow \left[\begin{array}{l} \mu_1 > \mu_2 \\ \begin{cases} \mu_1 = \mu_2 \\ \nu_1 \le \nu_2 \end{cases} \end{array}\right.,$$

$$(\mu_1,\nu_1) \ge_N (\mu_2,\nu_2) \Leftrightarrow \left[\begin{array}{l} \nu_1 < \nu_2 \\ \begin{cases} \nu_1 = \nu_2 \\ \mu_1 \ge \mu_2 \end{cases} \end{array}\right..$$

Some linguistic intuitionistic aggregation operators was proposed by using $\ge_M$, $\ge_N$ relations [16]. These operators are the simplest linguistic intuitionistic aggregations, which could be used to develop other operators for aggregating linguistic intuitionistic information.

In this paper, a new order relation on IS is proposed (a new relation is denoted by $\ge_3$. $\ge_M$, $\ge_N$ assigned to $\ge_1$, $\ge_2$ respectively). This implied from observation that: how a linguistic intuitionistic label great may depend on:

- How its membership component is greater than its non-membership one;

- How much information is contained in it.

For each $A=(s_i,s_j)\in IS$, these properties can be measured by $i-j$, $i+j$ which respectively called score and confidence of A.

Definition 3.3. For each $A=(a_i,a_j)$ in IS, score and confidence of A ($SC(A)$ and $CF(A)$ respectively) are define as follows

$$SC(A)=i-j,\ CF(A)=i+j.$$

Definition 3.4. For all A, B in IS, relation $\geq_3$ is defined as following

$$A\geq_3 B \Leftrightarrow \left[\begin{array}{l} SC(A)>SC(B) \\ \begin{cases} SC(A)=SC(B) \\ CF(A)\geq CF(B) \end{cases} \end{array}\right. .$$

Theorem 3.1. Relation $\geq_3$ is a total order relation.

Proof. It is easily seen that $\geq_3$ is reflexive and asymmetric. We now consider the transitivity and totality. Let A, B, C be arbitrary intuitionistic linguistic labels, we have:

● Transitivity: let us assume that $A\geq_3 B$ and $B\geq_3 C$. Then

$$\left[\begin{array}{l} SC(A)>SC(B) \\ \begin{cases} SC(A)=SC(B) \\ CF(A)\geq CF(B) \end{cases} \end{array}\right. \text{AND} \left[\begin{array}{l} SC(B)>SC(C) \\ \begin{cases} SC(B)=SC(C) \\ CF(B)\geq CF(C) \end{cases} \end{array}\right.$$

$$\Leftrightarrow \begin{cases} SC(A)>SC(B) \\ SC(B)>SC(C) \end{cases} \text{OR} \begin{cases} SC(A)>SC(B) \\ SC(B)=SC(C) \\ CF(B)\geq CF(C) \end{cases}$$

$$\text{OR} \begin{cases} SC(A)=SC(B) \\ CF(A)\geq CF(B) \\ SC(B)>SC(C) \end{cases} \text{OR} \begin{cases} SC(A)=SC(B) \\ CF(A)\geq CF(B) \\ SC(B)=SC(C) \\ CF(B)\geq CF(C) \end{cases}$$

$$\Rightarrow SC(A)>SC(C) \text{ OR } \begin{cases} SC(A)=SC(C) \\ CF(A)\geq CF(C) \end{cases}$$

$$\Rightarrow A\geq_3 C.$$

● Totality: There are four cases.

Case 1. $SC(A)>SC(B)$. In this case, $A\geq_3 B$.

Case 2. $SC(A)<SC(B)$. This implies $B\geq_3 A$.

Case 3. $\begin{cases} SC(A)=SC(B) \\ CF(A)\geq CF(B) \end{cases}$. We have $A\geq_3 B$.

Case 4. $\begin{cases} SC(A)=SC(B) \\ CF(A)<CF(B) \end{cases}$. This condition implies $B\geq_3 A$. +

Using this relation, we define max, min operators as the following:

$$\max(A_1,A_2,\ldots,A_m)=B_1$$

$$\min(A_1,A_2,\ldots,A_m)=B_m,$$

where $A_i\in IS$ for all i, $B_1=A_{\sigma(1)}$, $B_m=A_{\sigma(m)}$, σ is a permutation $\{1,2,\ldots,m\}\to\{1,2,\ldots,m\}$ such that $A_{\sigma(1)}\geq_3 A_{\sigma(2)}\geq_3 \ldots \geq_3 A_{\sigma(m)}$.

In order to convert linguistic intuitionistic labels to linguistic labels, we define $CV: IS\to S$ such that:

- $\begin{cases} SC(A)\geq SC(B) \\ CF(A)\geq CF(B) \end{cases} \Rightarrow CV(A)\geq CV(B), \forall A,B\in IS$;

- CV maps a linguistic label to itself (linguistic label s_i is identified with linguistic intuitionistic label (s_i,s_{n+1-i})):

$$CV((s_i,s_{n+1-i}))=s_i\ \forall s_i\in S.$$

Definition 3.5. For each $A=(s_i,s_j)$ in IS, we define

$$CV(A)=s_p,$$

where $p=\max\{i-\min\{j,n+1-i-j\},1\}$.

In the following theorem, we examine desiderative properties of CV.

Theorem 3.2. For all A, $B \in IS$, we have

(1) $CV(A) \in S$;

(2) $\begin{cases} SC(A) \geq SC(B) \\ CF(A) \geq CF(B) \end{cases} \Rightarrow CV(A) \geq CV(B)$;

(3) $A = (s_i, s_{n-i+1}) \Rightarrow CV(A) = s_i$.

Proof. Let us assume that $s_p = CV(A)$, and $s_q = CV(B)$, where $A = (s_i, s_j)$, and $B = (s_h, s_k)$. Then,

$p = \max\{i - \min\{j, n+1-i-j\}, 1\}$, and

$q = \max\{h - \min\{k, n+1-h-k\}, 1\}$.

(1) It is easily seen that $1 \leq p \leq n$, then $CV(A) \in S$.

(2) By $SC(A) \geq SC(B)$,

$$i - j \geq h - k. \quad (1)$$

By $SC(A) \geq SC(B)$, and $CF(A) \geq CF(B)$,

$\begin{cases} i - j \geq h - k \\ i + j \geq h + k \end{cases}$, or $\begin{cases} i - h \geq j - k \\ i - h \geq k - j \end{cases}$.

So, $i - h \geq 0$. Then

$$[i - (n+1-i-j)] - [h - (n+1-h-k)]$$
$$= 2(i-h) - (k-j) \geq (i-h) - (k-j) \geq 0$$
$$\Rightarrow i - (n+1-i-j) \geq h - (n+1-h-k). \quad (2)$$

By (1)-(2),

$$i - \min\{j, n+1-i-j\}$$
$$= \max\{i - j, i - (n+1-i-j)\}$$
$$\geq \max\{h - k, h - (n+1-h-k)\}$$
$$= h - \min\{k, n+1-h-k\}$$
$$\Rightarrow CV(A) \geq CV(B).$$

(3) If $A = (s_i, s_{n-i+1})$ and $s_p = CV(A)$,

$$p = \max\{i - \min\{j, n+1-i-(n+1-i)\}, 1\}$$
$$= \max\{i - \min\{j, 0\}, 1\} = i.$$

B. Linguistic Intuitionistic Fuzzy Relations

Linguistic intuitionistic fuzzy relation is defined in a similar way to intuitionistic fuzzy relation; however the correspondence of each pair of objects is given as a linguistic intuitionistic label.

Definition 3.6. Let X and Y be finite non-empty sets. A linguistic intuitionistic fuzzy relation R between X and Y is given by

$$R = \{\langle (x,y), \mu_R(x,y), \nu_R(x,y) \rangle | (x,y) \in X \times Y\},$$

where, for each $(x,y) \in X \times Y$:

- $(\mu_R(x,y), \nu_R(x,y)) \in IS$;

- $\mu_R(x,y)$ and $\nu_R(x,y)$ define linguistic membership degree and linguistic non-membership degree of (x,y) in the relation R, respectively.

The set of all linguistic intuitionistic fuzzy relations is denoted by $LIFR(X \times Y)$. We denote the pair $(\mu_R(x,y), \nu_R(x,y))$ by $R(x,y)$. So, $\langle (x,y), \mu_R(x,y), \nu_R(x,y) \rangle = \langle (x,y), R(x,y) \rangle$.

There are some ways to define linguistic membership degree and linguistic non-membership degree in linguistic intuitionistic fuzzy relations. The following is an example:

Example. Experts use linguistic labels to access the interconnection R between two objects x and y. There are assessments voting for satisfaction of (x,y) into R, the remainders vote for dissatisfaction of (x,y) into R. Aggregating the first group of assessments, we obtain linguistic membership degree; aggregating the second one, we obtain linguistic membership degree (for example, use fuzzy collective solution [20]).

In the following, max–min composition of two linguistic intuitionistic fuzzy relations is defined.

Definition 3.7. Let $R \in LIFR(X \times Y)$, $P \in LIFR(Y \times Z)$. Max–min composition o between R and P is defined by

$$P \circ R = \{\langle (x,z), P \circ R(x,z) \rangle | (x,z) \in X \times Z\},$$

where $\mu_{P \circ R}(x,z) = \max_y \{\min[R(x,y), P(y,z)]\}$,

$$\forall (x,z) \in X \times Z.$$

C. Application in Medical Diagnosis

In this section, we present an application of linguistic intuitionistic fuzzy relation in Sanchez's approach for medical diagnosis [12-13]. In a given pathology, suppose that P is the set of patients, S is the set of symptoms, and D is the set of diagnoses.

Now let us discuss linguistic intuitionistic fuzzy medical diagnosis. The methodology mainly involves with the following four steps:

Step 1 Determination of symptoms.

In this step, the interconnection between each patient and each symptom is given by a linguistic membership grade and a linguistic non-membership grade. All such interconnections form linguistic intuitionistic fuzzy relation Q between P and S. Here, the linguistic membership grades and the linguistic non-membership grades could be collected by examination of doctors.

Step 2 Formulation of medical knowledge based on linguistic intuitionistic fuzzy relations.

Analogous to the Sanchez's notion of "Medical Knowledge" we define "Linguistic Intuitionistic Medical Knowledge" as a linguistic intuitionistic fuzzy relation R between the set of symptoms S and the set of diagnoses D which expresses the membership grades and the non-membership grades between symptoms and diagnosis. This relation can be obtained by from medical experts or some training processes.

Step 3 Determination of diagnosis using the composition of linguistic intuitionistic fuzzy relations.

In this step, relation T is determined as composition of the relations Q (step 1) and R (step 2). So, T is the relation between P and D.

Step 4 Using the mapping CV (definition 3.5), converting T (step 3) into linguistic fuzzy relation SR.

For each patient p and diagnosis d, if $SR(p,d)$ is greater than or equal to the median value of S, it is stated that p suffers from d.

Let us consider a case study, adapted from De, Biswas Roy [8], where

- The set of patients is $\mathrm{P} = \{p_1, p_2, p_3, p_4\}$,
- The set of symptoms is

$$\mathrm{S} = \{Temperature, Headache, Stomach\ Pain, Cough, Chest\ Pain\},$$

- and the set of diagnoses is

$$\mathrm{D} = \{Viral\ Fever, Malaria, Typhoid, Stomach\ problem, Heart\ problem\}.$$

In this example, intuitionistic label set IS is constructed using label set:

$$S = \{s_1 = none, s_2 = very\ low, s_3 = low, s_4 = lightly\ low, s_5 = medium, s_6 = lightly\ high, s_7 = high, s_8 = very\ high, s_9 = perfect\}.$$

The linguistic intuitionistic fuzzy relations $Q \in LIFR(\mathrm{P} \times \mathrm{S})$ and $R \in LIFR(\mathrm{S} \times \mathrm{D})$ are hypothetical given as in table II and table III. The linguistic intuitionistic fuzzy relation T (table IV) and linguistic fuzzy relation S_R (table V) are obtained as follows:

- $T = R \circ Q$, where o is max-min composition (definition 3.7). For example,

$T(p_2, Typhoid)$

TABLE II
LINGUISTIC INTUITIONISTIC RELATION BETWEEN PATIENTS AND SYMPTOMS

Q	TEMPERATURE	HEADACHE	STOMACH PAIN	COUGH	CHEST PAIN
p_1	(s_8, s_1)	(s_6, s_1)	(s_2, s_7)	(s_6, s_1)	(s_1, s_6)
p_2	(s_1, s_7)	(s_4, s_4)	(s_6, s_1)	(s_1, s_6)	(s_1, s_7)
p_3	(s_8, s_1)	(s_8, s_1)	(s_1, s_7)	(s_2, s_7)	(s_1, s_4)
p_4	(s_5, s_1)	(s_5, s_3)	(s_3, s_4)	(s_6, s_1)	(s_2, s_3)

TABLE III
LINGUISTIC INTUITIONISTIC RELATION BETWEEN SYMPTOMS AND DIAGNOSES

R	VIRAL FEVER	MALARIA	TYPHOID	STOMACH PROBLEM	CHEST PROBLEM
TEMPE-RATURE	(s_5, s_1)	(s_6, s_1)	(s_2, s_2)	(s_2, s_6)	(s_1, s_7)
HEAD-ACHE	(s_1, s_7)	(s_4, s_4)	(s_6, s_1)	(s_1, s_6)	(s_1, s_7)
STOMACH PAIN	(s_2, s_6)	(s_1, s_7)	(s_1, s_5)	(s_7, s_1)	(s_1, s_7)
COUGH	(s_4, s_3)	(s_6, s_1)	(s_2, s_6)	(s_1, s_7)	(s_2, s_7)
CHEST PAIN	(s_2, s_7)	(s_1, s_8)	(s_2, s_6)	(s_1, s_7)	(s_8, s_1)

TABLE IV
LINGUISTIC INTUITIONISTIC RELATION BETWEEN PATIENTS AND DIAGNOSES

T	VIRAL FEVER	MALARIA	TYPHOID	STOMACH PROBLEM	CHEST PROBLEM
p_1	(s_5, s_1)	(s_6, s_1)	(s_6, s_1)	(s_2, s_6)	(s_2, s_7)
p_2	(s_2, s_6)	(s_4, s_4)	(s_4, s_4)	(s_6, s_1)	(s_1, s_6)
p_3	(s_5, s_1)	(s_6, s_1)	(s_6, s_1)	(s_2, s_6)	(s_1, s_4)
p_4	(s_5, s_1)	(s_6, s_1)	(s_5, s_3)	(s_3, s_4)	(s_2, s_3)

$$= \max \left\{ \begin{array}{l} \min\{Q(p_2, Temperature), R(Temperature, Typhoid)\}, \\ \min\{Q(p_2, Headache), R(Headache, Typhoid)\}, \\ \min\{Q(p_2, Stomach\ Pain), R(Stomach\ Pain, Typhoid)\}, \\ \min\{Q(p_2, Cough), R(Cough, Typhoid)\}, \\ \min\{Q(p_2, Chest\ Pain), R(Chest\ Pain, Typhoid)\} \end{array} \right\}$$

$$= \max\{(s_1, s_7), (s_4, s_4), (s_1, s_5), (s_1, s_6), (s_1, s_7)\} = (s_4, s_4).$$

- Using mapping CV (definition 3.5), T is converted to linguistic fuzzy relation S_R. For example,

$$S_T(p_2, Typhoid) = CV(T(p_2, Typhoid))$$

$$= CV(T(p_2, Typhoid)) = CV((s_4, s_4))$$

$$= s_{\max\{4-\min\{2,9+1-4-4\},1\}} = s_{\max\{4-\min\{2,2\},1\}}$$

$$= s_{\max\{4-2,1\}} = s_{\max\{2,1\}} = s_2.$$

For each patient p and diagnosis d, if $S_R(p, d) \geq s_5$ (s_5 is the median value of the label set S), p suffers from d. From table V, it is obvious that, if the doctor agrees, p_1, p_3 and p_4 suffer from Malaria, p_1 and p_3 suffer from Typhoid whereas p_2 faces Stomach problem.

4. Conclusion

In this paper, linguistic intuitionistic fuzzy relation is introduced. Max - min composition of linguistic intuitionistic fuzzy relations is defined using a new order relation on intuitionistic label set. New notions are applied in medical diagnosis. This gives a flexible and simple solution for medical diagnosis problem in linguistic and intuitionistic environment.

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