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A general model of Fractional Frequency Reuse: Modelling and Performance Analysis

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Abstract

Fractional Frequency Reuse (FFR) is a promising to improve the spectrum efficiency in the LongTerm Evolution (LTE) cellular network. In the literature, various research works have been conducted to evaluate the performance of FFR. However, the presented analytical approach only dealt with the special cases in which the users are divided into 2 groups and only two power levels are utilised. In this paper, we consider a general case of FFR in which the users are classified into Δ groups and each group is assigned a serving power level. The mathematical model of the general FFR is presented and analysed through a stochastic geometry approach. The derived analytical results in terms of average coverage probability can covered all the related well-known results in the literature.

Keywords: Fractional Frequency Reuse, LongTerm Evolution, coverage probability, stochastic geometry.

1. Introduction

In recent years, there has been a rapid rise in the number of mobile users and mobile data traffic. According to Cisco report [1], the number of mobile users has a 5-fold growth over the past 15 years. In 2015 more than a half of a million devices have joined the cellular networks. It is predicted that the number of mobile users will reach 5.5 billion by 2020 which represents 70% of the global population. This will make mobile data traffic experience eight-fold over the next five years. Therefore, the requirement of spectral efficiency improvement is a big challenge for the network designers and operators.

One of the most popular to improve spectral efficiency relates to frequency resource allocation in which all Base Stations (BSs) are allowed to operate on all Resource Blocks (BSs). It is reminded that in Long Term Evolution (LTE) network, each RB is defined as having a time duration of 0.5ms and a bandwidth of 180kHz made up of 12 sub-carriers with a sub-carrier spacing of

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15kHZ. Due to sharing RBs between BSs, InterCell Interference (ICI) which is caused by using the same RB at adjacent cells at the same time becomes a main negative factor to limit the network performance. Therefore, Fractional Frequency Reuse (FFR) algorithms have been introduced to control the reuse of frequency [2].

The basic idea of FFR algorithm is to divide the active users as well as the allocated RBs into some groups so each group of users is served by a specific group of RBs.

As recommendations of 3GPP [3, 4, 5], the BS can utilise a lower power level to serve the user with better wireless channel, an a higher power level to sever other users. By this way, the main benefits are expected to achieve as follows:

- Reduce the power consumption of the BSs. Some users with good communication links such as low propagation path loss, low fading can obtain their desired performance with low power levels. Thus, the BSs do not need to use high power levels to serve those users.
- Improve system performance. It is obvious that when a BS cuts its transmit power off, its interfering power at the adjacent cell will be reduced. Thus, the system performance can be improved.

In the literature, there are a lot of research works on modelling and performance analysis of FFR in LTE networks by utilizing the simulators such as [2, 6, 7, 8] or stochastic geometry models such as [9, 10, 11]. However, these works only considered two groups of users and thus only two power levels were utilised. In a real network, the users as well as RBs can be partitioned into more than two groups. For example, a macro cell with radius from 1 - 20 km can cover a huge area of up to 400 km^2. Thus, the users associated with that macro cell experiences a wide range of SINR and consequently they should be classified in more than two groups to achieve better network performance as well as save the power consumption of BSs.

Hence in this paper, we consider the FFR algorithm in which the users and RBs are classified into Δ groups ($\Delta \geq 2$). Thus, N power levels are deployed, in which each user group is served by a group of RB with a specific power level. Figure 1 is an example of the proposed model with frequency reuse $\Delta = 3$.

The operational discipline of the system model can be described as follows:

- Every $\Delta = 3$ cells use the same frequency reuse pattern.
- The users are classified into 3 groups by two SINR thresholds. There power levels are denoted by P_1, P_2 and P_3.
- The resource and power allocations are presented in Table 1

	Cell 1	Cell 2	Cell 3
RB group 1	P_3	P_2	P_1
RB group 2	P_2	P_1	P_3
RB group 3	P_1	P_2	P_3

Table 1. Power allocation in the case of $\Delta = 3$

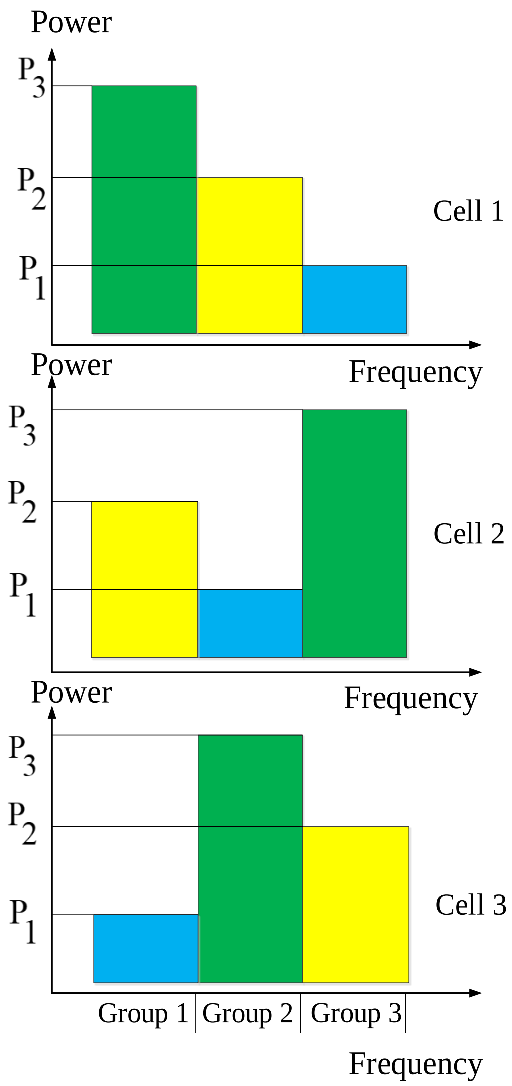

Figure 1. a proposed FFR algorithm with $\Delta = 3$

2. System model

We consider a single tier cellular network in which the locations of BSs follows a spatial Poison Point Process (PPP) with mean λ. The user prefers a connection with the nearest BS.

According to 3GPP recommendations at [12, 13], the operation of FFR includes two phases, called establishment phase and communication phase. The detail of these phases are described as follows:

Establishment phase. The users measure and report the received SINRs on the downlink control channels [12, 13] for user classification purpose. Every BS is continuously transmitting downlink control information, and subsequently each control channel experiences the ICI from all adjacent BSs. Furthermore, since all BSs are assumed to transmit on the control channels at the same power, the ICI of the measured SINR during this phase is given by

$$I_0 = \sum_{j \in \theta} P g_j^{(o)} r_j^{-\alpha} \quad (1)$$

where $g_j^{(o)}$ and r_j are the channel power gain and distance between BS j and the user, respectively.

The reported SINR on the control channel is given by

$$SINR = \frac{P g^{(o)} r^{-\alpha}}{\sigma^2 + \sum_{j \in \theta} P g_j^{(o)} r_j^{-\alpha}} \quad (2)$$

in which g and r is the channel power gain and the distance from the user to its serving BS.

In stead of assuming that there are only two groups of users, we classified users into Δ groups by $\Delta - 1$ SINR thresholds. The user j is assigned to group j if its downlink SINR on the control channel satisfy the following condition

$$T_{n-1} < SINR < T_n \quad (3)$$

in which T_n is the SINR threshold j, $T_0 = 0$,

$T_\Delta = \infty$, and $T_{j-1} < T_n$ for $\forall\ 0 < j \leq \Delta$.

Communication phase. We denote the transmit power used to serve users in group j is P_j. Since the high power levels are used to serve users with the lower SINR on the control channel, $P_{n-1} < P_j$ for $\forall\ 1 < j \leq N$. We denote the ratio between the power levels and the lowest power level P_1 is $\phi_j = P_j/P_1$.

Due to sharing the RBs between cells, each user experiences ICI from all neighbouring cells. The total ICI power at the typical user is given by

$$I = \sum_{k=1}^{\Delta} P_k \sum_{j \in \theta_k} g_j r_j^{-\alpha} \tag{4}$$

in which θ_k is the set of interfering BSs transmitting at P_j power level. The density of BSs in θ_k is $\frac{\lambda}{\Delta}$.

Equation 4 can be considered as the general case of the well-known FFR algorithm modelling in the literature. For examples:

- When $\Delta = 1$, Equation 4 degrades into

  $$I = \sum_{j \in \theta} P g_j r_j^{-\alpha} \tag{5}$$

  In Equation 5, θ consists of all adjacent BSs. This equation has been found in the literature such as [14, 15].

- When only two power levels are deployed (only one SINR threshold is required): for example group 1 is served by transmit power P_1 and $\Delta - 1$ remaining groups are served by transmit power P_2, Equation 4 degrades into

  $$I = \sum_{j \in \theta_1} P_1 g_j r_j^{-\alpha} + \sum_{k=1}^{\Delta-1} \sum_{j \in \theta_k} P_2 g_j r_j^{-\alpha} \tag{6}$$

  Due to the thinning properties of PPP [16], each BS in θ_1 is distributed independently to any BS in θ_k ($j \neq 1$). Therefore, Equation 6 is rewritten as

  $$I = \sum_{j \in \theta_1} P_1 g_j r_j^{-\alpha} + \sum_{j \in \theta_0} P_2 g_j r_j^{-\alpha} \tag{7}$$

  in which the density of BSs in θ_1 and θ_2 are Δ/λ and $(\Delta - 1)\lambda/\Delta$ respectively.

  Equation 7 is exactly the ICI of Soft FR which has derived in [17].

The reported SINR on the data channel during the communication phase is given by

$$SINR' = \frac{P g r^{-\alpha}}{\sum_{k=1}^{\Delta} P_k \sum_{j \in \theta_k} g_j r_j^{-\alpha}} \tag{8}$$

in which g and r is the channel power gain and the distance from the user to its serving BS.

3. Performance Evaluation

In this section, we derive the average coverage probability of the typical user, which can be classified into one of Δ groups.

At a given time slot, the user at a distance r from its serving BS is assigned to group j if its downlink SINR satisfies Equation 3. The corresponding probability is $P(T_{n-1} < SINR < T_n)$.

The user in group j is under the network coverage if its SINR during the communication phase, denoted by $SINR'$, is greater than the coverage threshold $\hat{T}$. Thus, the coverage probability is $P(SINR' > \hat{T})$.

Therefore, the probability in which the typical user is under the network coverage at a given time slot is given by

$$P_c = \sum_{n=1}^{\Delta} P(T_{n-1} < SINR < T_n) \qquad P(SINR_n > \hat{T}) \tag{9}$$

It is reminded that the coverage probability in Equation 9 is a function of random variables such as channel power gain g, g_j, distance from the user to other BSs. Thus, to obtain the average coverage probability of the typical user, the expected value of P_c should be computed. Therefore, the average coverage probability of the user in the network is defined as following equation:

$$\mathcal{P}(\hat{T}) = \sum_{n=1}^{\Delta} \mathbb{E}\Bigg(P(T_{n-1} < SINR < T_n) \qquad P(SINR_n > \hat{T})\Bigg) \tag{10}$$

Using the definition of SINR in Equation 2,

$$\begin{aligned} &P(T_{n-1} < SINR < T_n) \\ &= P\left(T_{n-1} < \frac{g^{(o)} r^{-\alpha}}{\sum_{j\in\theta} g_j^{(o)} r_j^{-\alpha}} < T_n\right) \\ &= P\left(T_{n-1} \sum_{j\in\theta} g_j^{(o)} \frac{r^{\alpha}}{r_j^{\alpha}} < g^{(o)} < T_n \sum_{j\in\theta} g_j^{(o)} \frac{r^{\alpha}}{r_j^{\alpha}}\right) \\ &\overset{(a)}{=} \prod_{j\in\theta} \exp\left(-T_{n-1} g_j^{(o)} r_j^{-\alpha} r^{\alpha}\right) \\ &\qquad - \prod_{j\in\theta} \exp\left(T_n g_j^{(o)} r_j^{-\alpha} r^{\alpha}\right) \end{aligned} \tag{11}$$

in which (a) due to $g^{(o)}$ has a exponential distribution.

Similarity, using the definition of $SINR'$ in Equation 8, we have

$$\begin{aligned} &P(SINR_n > \hat{T}) \\ &= P(\frac{P_n g r^{-\alpha}}{\sum_{k=1}^{\Delta} P_k \sum_{j\in\theta_k} g_j r_j^{-\alpha}} > \hat{T}) \\ &= P\left(g > \hat{T} \sum_{k=1}^{\Delta} \frac{P_k}{P_n} \sum_{j\in\theta_k} g_j r_j^{-\alpha} r^{\alpha}\right) \\ &\overset{(b)}{=} \prod_{k=1}^{\Delta} \prod_{j\in\theta_k} \exp\left(-\hat{T} \frac{P_k}{P_n} g_j r_j^{-\alpha} r^{\alpha}\right) \end{aligned} \tag{12}$$

where (b) due to $g^{(o)}$ is a exponential random variable.

Substituting Equations 11 and 12 into Equation 10, the average coverage probability $\mathcal{P}(\hat{T})$is given by

$$\begin{aligned} &\sum_{n=1}^{\Delta} \mathbb{E}\Bigg[\Bigg(\prod_{k=1}^{\Delta} \prod_{j\in\theta_k} \exp\left(-\hat{T} \frac{P_k}{P_n} g_j r_j^{-\alpha} r^{\alpha}\right)\Bigg) \\ &\qquad \times \Bigg(\prod_{j\in\theta} \exp\left(-T_{n-1} g_j^{(o)} r_j^{-\alpha} r^{\alpha}\right) \\ &\qquad - \prod_{j\in\theta} \exp\left(T_n g_j^{(o)} r_j^{-\alpha} r^{\alpha}\right)\Bigg)\Bigg] \end{aligned} \tag{13}$$

Since all channel power gains are independent exponential random variables whose the Moment Generating Function (MGF) is $M_X = E[e^{-sx}] = \frac{1}{1+s}$, taking the expected value of Equation with respect to

$g_j^{(o)}$ and g_j, $\mathcal{P}(\hat{T})$ is obtained by

$$\sum_{n=1}^{\Delta}\mathbb{E}\left[\prod_{k=1}^{\Delta}\prod_{j\in\theta_k}\frac{1}{1+\hat{T}\frac{P_k}{P_n}\frac{r^\alpha}{r_j^\alpha}}\prod_{j\in\theta}\frac{1}{1+T_{n-1}\frac{r^\alpha}{r_j^\alpha}}\right] - \sum_{n=1}^{\Delta}\mathbb{E}\left[\prod_{k=1}^{\Delta}\prod_{j\in\theta_k}\frac{1}{1+\hat{T}\frac{P_k}{P_n}\frac{r^\alpha}{r_j^\alpha}}\prod_{j\in\theta}\frac{1}{1+T_{n}\frac{r^\alpha}{r_j^\alpha}}\right] \tag{14}$$

Evaluating the fist element with notice that θ_k is a subset of θ, we divide θ into Δ independent subsets θ_k with the densities of BSs are λ_k/Δ. Thus, the first element in Equation 14 can be rewritten as follows

$$E_1 = \sum_{n=1}^{\Delta}\mathbb{E}\left[\prod_{k=1}^{\Delta}\prod_{j\in\theta_k}\left(\frac{1}{1+\hat{T}\frac{P_k}{P_n}\frac{r^\alpha}{r_j^\alpha}}\frac{1}{1+T_{n-1}\frac{r^\alpha}{r_j^\alpha}}\right)\right]$$

Employing the properties of the Probability Generating Function [18], we obtain

$$E_1 = \sum_{n=1}^{\Delta}\mathbb{E}\left[\prod_{k=1}^{\Delta} e^{-\frac{2\pi\lambda_k}{\Delta}\int_r^\infty\left(1-\frac{1}{1+\hat{T}\frac{P_k}{P_n}\frac{r^\alpha}{r_j^\alpha}}\frac{1}{1+T_{n-1}\frac{r^\alpha}{r_j^\alpha}}\right)r_j dr_j}\right]$$

Using a change of variable $y = (r_j/r)^2$, E_1 can be rewritten as follows

$$E_1 = \sum_{n=1}^{\Delta}\mathbb{E}\left[\prod_{k=1}^{\Delta} e^{-\frac{2\pi\lambda_k r^2}{\Delta}\int_1^\infty\left(1-\frac{1}{1+\hat{T}\frac{P_k}{P_n}y^{-\alpha/2}}\frac{1}{1+T_{n-1}y^{-\alpha/2}}\right)dy}\right]$$

Taking the expected value with respect to r, E_1 is given by

$$2\pi\lambda\sum_{n=1}^{\Delta}\int_0^\infty re^{-\pi\lambda_k r^2}e^{-\frac{\pi r^2}{\Delta}\sum_{k=1}^{\Delta}\lambda_k\upsilon_n(T_{n-1},\hat{T},P_k)}dr$$

in which $\upsilon_n(T_{n-1},\hat{T},P_k) = \int_1^\infty\left(1-\frac{1}{1+\hat{T}\frac{P_k}{P_n}y^{-\alpha/2}}\frac{1}{1+T_{n-1}y^{-\alpha/2}}\right)dy$

Similarly, the second element of Equation 14 is given by

$$E_2 = 2\pi\lambda\sum_{n=1}^{\Delta}\int_0^\infty re^{-\pi\lambda r^2}e^{-\frac{\pi r^2}{\Delta}\sum_{k=1}^{\Delta}\lambda_k\upsilon_n(T_n,\hat{T},P_k)}dr$$

Substituting E_1 and E_2 into Equation 14 and employing a change of variable in which $y = \pi\lambda r^2$, the average coverage probability $\mathcal{P}(\hat{T})$ is given by

$$\mathcal{P}(\hat{T}) = \sum_{n=1}^{\Delta}\frac{1}{1+\frac{1}{\Delta}\sum_{k=1}^{\Delta}\frac{\lambda_k}{\lambda}\upsilon_n(T_{n-1},\hat{T},P_k)} - \sum_{n=1}^{\Delta}\frac{1}{1+\frac{1}{\Delta}\sum_{k=1}^{\Delta}\frac{\lambda_k}{\lambda}\upsilon_n(T_{n},\hat{T},P_k)} \tag{15}$$

Equation 15 provides the mathematical expression of the average coverage probability of the typical user in LTE network using FFR with reuse factor Δ in which users are classified into Δ user groups. This result can be considered as the general form of the published results in the literature. Take two special cases, $\Delta = 1$ and $\Delta = 3$, for example

Special case 1: $\Delta = 1$

In this case, $T_0 = 0$ and $T_1 = \infty$, then $\upsilon_n(0,\hat{T},P_k) = \int_1^\infty\left(1-\frac{1}{1+\hat{T}\frac{P_k}{P_n}y^{-\alpha/2}}\right)dy$ and $\upsilon_n(\infty,\hat{T},P_k) = 0$.

The average coverage probability is given by

$$\mathcal{P}(\hat{T}) = \sum_{n=1}^{\Delta} \frac{1}{1 + \upsilon_n(T_{n-1}, \hat{T}, P_k)} \quad (16)$$

The expression in Equation 16 is the well-known result on the average coverage probability of the typical user in LTE network with frequency reuse factor $\Delta = 1$.

3.1. Special case 2: Only two power levels are deployed

This model is usually called Soft Frequency Reuse [19] in which the users and RBs are divided into Δ equal groups. Using the result in Equation 15 with $T_0 = 0, T_1, T_m = \infty\ \forall m \geq 2$ and $P_m = P_n\ \forall m, n > 2$, we obtain

$$\begin{aligned}\mathcal{P}(\hat{T}) = & \frac{1}{\left[\begin{array}{l}1 + \frac{\Delta-1}{\Delta}\upsilon_1(0, \hat{T}, P_1) \\ \qquad + \frac{1}{\Delta}\upsilon_1(0, \hat{T}, P_1)\end{array}\right]} \\ & + \frac{1}{\left[\begin{array}{l}1 + \frac{\Delta-1}{\Delta}\upsilon_2(T_1, \hat{T}, P_1) \\ \qquad + \frac{1}{\Delta}\upsilon_2(T_1, \hat{T}, P_2)\end{array}\right]} \\ & - \frac{1}{\left[\begin{array}{l}1 + \frac{\Delta-1}{\Delta}\upsilon_1(T_1, \hat{T}, P_1) \\ \qquad + \frac{1}{\Delta}\upsilon_1(T_1, \hat{T}, P_2)\end{array}\right]}\end{aligned} \quad (17)$$

The corresponding result for Soft Frequency Reuse algorithm has been found in [17].

4. Simulation and Discussion

Figure 2 presents the comparison between the simulation and analytical results with different values of path loss coefficient α and coverage threshold $\hat{T}$. As shown in

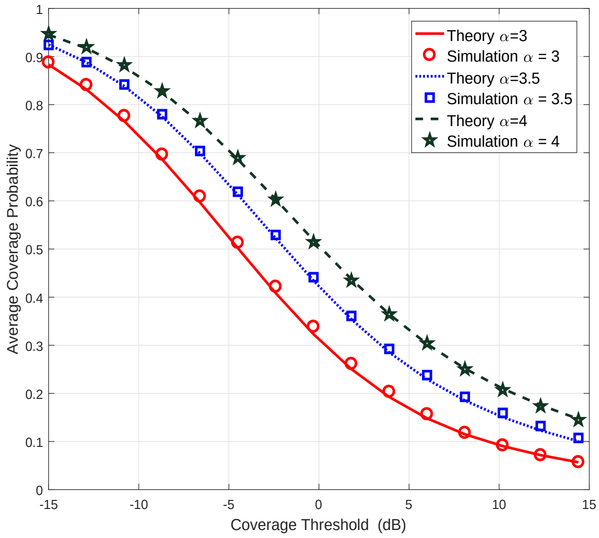

Figure 2. Comparison between simulation and analytical results

Figure 2, the Monte Carlo simulation results perfectly match with the analytical results that can confirm the accuracy of the analytical approach.

As indicated in Figure 2, the average coverage probability of the typical user increases with α. This conclusion also has been found in the literature and can be explained as follows:

- Since the user is assumed to associate with the nearest BS. The distance from the user to the interfering BSs must be greater than that from user the serving BS.
- The path loss is proportional to the path loss coefficient and the distance. Hence, when the path loss exponent increases, the interfering signals experience higher path loss than the serving signal. In other words, SINR and consequently average

coverage probability increase with the path loss exponent α.

Figure 3 compares the average coverage probability of the typical user with different values of δ and SINR threshold. The selection of parameters are as the following table: It is

	T_1	T_2	T_3
$\Delta = 2$	-10 (dB)		
$\Delta = 3$	-10 (dB)	0 (dB)	
$\Delta = 4$	-10 (dB)	0 (dB)	10 (dB)

Table 2. Analytical parameters of Figure 3

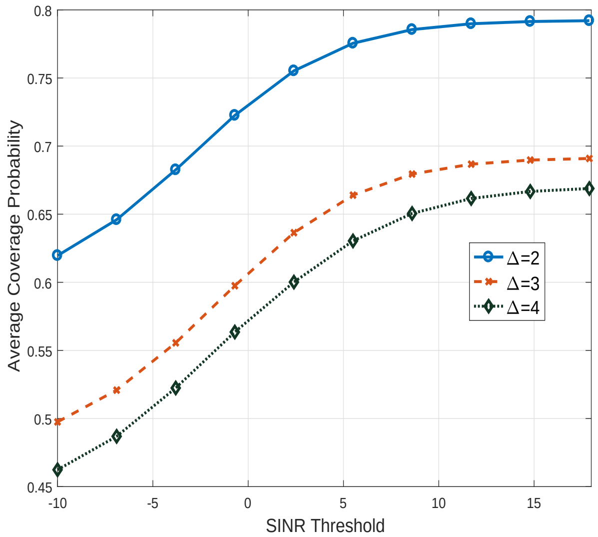

Figure 3. Comparison average coverage probability with different values of Δ

assumed that all users in Group 1 of these cases have the same serving power. The serving power of the adjacent group is reduced by 3 times. From Table 2, it is observed that the total energy that used by the BSs to serve the associated users increases with Δ. For example, the BSs in the case of $\Delta = 2$ consume more energy than that in the case $\Delta = 3$.

It is observed that the average coverage probability reduces when δ increases. This phenomenon is reasonable since the user achieves the higher performance with high serving power. However, in order to compare the performance of frequency reuse algorithms, various of parameters and scenarios should be considered [20].

5. Conclusion

In this paper, the general model of FFR in the LTE network was modelled and analysed under Rayleigh fading environment in which the BSs are distributed according to a spatial Poisson process. Instead of assuming that there are only two power levels are used to serve the associated user, this paper considered Δ power levels in which each power level is utilised to serve a specific user group. The analytical results which are verified by Monte Carlo simulation can be considered as the general expressions of the typical user performance since they contains all the related results in the literature.

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