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Selection of Appropriate Number of CRs in Cooperative Spectrum Sensing over Suzuki Fading

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Abstract

With the rapid development of wireless communications, the radio spectrum is becoming scarce. However, researchers have shown that many portions of licensed spectrum unused for significant periods of time. Recently, cognitive radio has been proposed as a very effective mechanism which allows Cognitive Radio Users (CRs) to utilize the idle unused licensed bands. The main challenge for a CR is to detect the existence of Primary User (PU) in order to minimize the interference to it. In this paper, we study the cooperative spectrum sensing under Suzuki composite fading channel which is the mixture of Rayleigh fading channel and Log-normal shadowing channel. Besides, we also concentrate on finding the minimum number of CRs taking part in the collaborative spectrum sensing to avoid the overhead to the network but still guarantee the sensing performance through calculations and numerical results. Our analysis and simulation results suggest that collaboration may improve sensing performance significantly.

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Keywords: Suzuki fading, composite fading, energy detection, cooperative spectrum sensing, cognitive radio.

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1. Introduction

In recent years, with the rapid development of science and technology, the number of portable digital assistants (PDAs), also known as handhold PCs, such as smartphone, tablet, etc., has been increasing suddenly. New technologies enable these devices to acquire data at a high rate from 1 to 10 Mbps. In the next few years, the rate is going to climb up to 100 Mbps and perhaps exceeds the rate of 1 Gbps in the following decades. OFDM and MIMO techniques enhanced the spectrum efficiency to about 4 b/s/Hz and can achieve 8 b/s/Hz or higher in the future, only 8 times larger than the spectrum efficiency of GSM and CDMA networks (1 b/s/Hz). However, multimedia services require a data rate of 10 MHz, (i.e over 100 fold increase compare to the rate of traditional voice services) and leads to the lack of bandwidth at the licensed frequency spectrum. To solve the spectrum scarcity problem, Cognitive Radio has been proposed as a promising technology for the next generations of wireless communications such as 4G or 5G [1].

In order to guarantee that the operation of the PU is not affected, the secondary users, or CRs, must have the ability of sensing the presence of active primary users, and this process is called spectrum sensing [2]. Spectrum sensing is the first step for CRs to implement the cognitive radio system. This step indicates the states of the frequency band of the primary system so that the CRs decide opportunistically to access the temporarily unused licensed band. Unfortunately, multipath fading (eg. Rayleigh fading) and shadowing are the causes that obstruct the sensing

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ability of the individual CRs. To solve such problems, multiple CRs can cooperate with each other to achieve an enhanced spectrum sensing performance [3, 4, 5]. In collaborative spectrum sensing, each CR processes the received signal to make a decision (a binary decision) on the PU activity, and the individual decision is reported to a Fusion Center, or FC, over a reporting channel. The reporting channel may have a narrow bandwidth [6]. The mission of the FC is to analyze and fuse the coming signals from CRs to derive a global decision on the presence of the PU. The fusion rule at the FC is based on the k-out-of-n rule which can be *OR*, *AND*, or *MAJORITY* rule.

In recent years, many researchers have been interested in the affects of these fadings on the sensing performance of a CR network through energy detection technique [6, 7]. However, the effect of composite Rayleigh - Lognormal fading, which is also known as Suzuki fading [8], on the spectrum sensing capacity still has not been concerned much.

Besides, we are also interested in investigating the affect of the number of CRs participating in collaborative spectrum sensing on the sensing performance. Previous works [4, 5, 7] showed that the spectrum sensing performance was improved significantly when the number of CRs increased. In fact, when too many CRs participating in the sensing process, a very large amount of sensing information is sent from the CRs to the FC and therefore, at the FC, it wastes more time processing that information. Moreover, the more CRs participate in cooperative spectrum sensing, the more overhead the network have to suffer. A question arises: What is the required number of CRs to avoid wasting network resources as well as overhead in network but still guarantee the detection performance? To answer this question, we also derived a formula for calculating the most suitable number of CRs so that the sensing performance is maximum.

The remainder of the paper is organized as follows. In Section 2, the system model for a Cognitive radio network and the energy detection are briefly introduced. Section 3 discusses the local spectrum sensing over Rayleigh fading and Lognormal shadowing channels in order to construct the formula for local spectrum sensing over Suzuki channel as well as shows the limitations of local spectrum sensing. Then, the cooperative spectrum sensing is investigated and the appropriate number of CRs participating in the cooperative sensing is found out in Section 4. Finally, Section 5 concludes the paper.

2. System Model

Consider a cognitive radio network with N CRs and an FC, as shown in Figure 1. Assume that each CR is equipped with an energy detector and can perform local spectrum sensing independently. Each CR makes its own observation based on the received signal, that is, noise only or signal plus noise. Hence, the spectrum sensing problem can be considered as a binary hypothesis testing problem defined as,

$$x(t) = \begin{cases} n(t), & H_0(whitespace) \\ hs(t) + n(t), & H_1(occupied) \end{cases}$$

where $x(t)$ is the signal received by the CR, $s(t)$ is the PU's transmitted signal, $n(t)$ is the Additive White Gaussian Noise (AWGN) and h is the amplitude gain of the channel. The signal-to-noise ratio (SNR) is defined as $\gamma = \frac{P}{N_0 W}$ with P and N_0 being the power of the primary signal received at the secondary user and the one-sided noise power spectrum density, respectively, and W being the bandwidth of an ideal bandpass filter which is referred in Figure 2 below.

Figure 2 describes the block diagram of an energy detector. The received signal is first pre-filtered by an input bandpass filter whose center frequency is f_s, and bandwidth of interest is W to eliminate the out-of-band noise. The filter is followed by a squaring device to measure the received energy and an integrator which determines the observation interval, T. The output of the integrator is then normalized by $N_0/2$ before being passed to a threshold device in which the normalized output, Y, is compared to

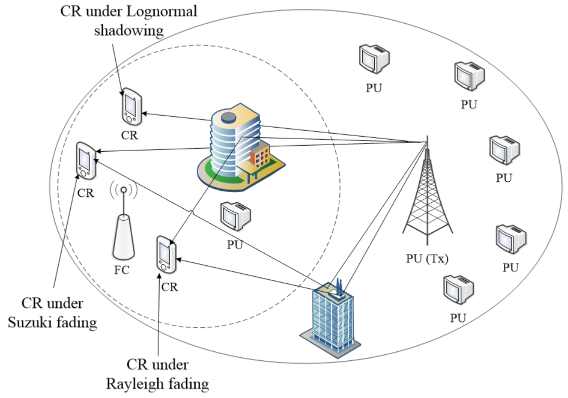

Fig. 1: System model of Cognitive Radio network [9].

a threshold value, λ, to decide whether the signal (i.e. PU's signal) is present (H_0) or absent (H_1).

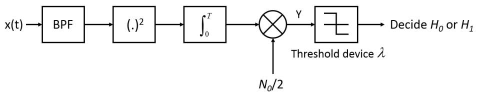

Fig. 2: Block diagram of energy detection [4].

For simplicity, we assume that the time-bandwidth product, TW, is always an integer number which is denoted by m. According to the work of Urkowitz [10], the output of the integrator, Y is the sum of squares of m Gaussian random variables and it follows a chi-square distribution,

$$Y \sim \begin{cases} \chi^2_{2TW}, & H_0 \\ \chi^2_{2TW}(2\gamma), & H_1 \end{cases}$$

where χ^2_{2TW} and $\chi^2_{2TW}(2\gamma)$ denote central and non-central chi-square distributions, respectively, each has $2m$ degrees of freedom, and a non-centrality parameter of 2γ for latter distribution. The energy detection process can be briefly expressed by equation,

$$Y \underset{H_0}{\overset{H_1}{\gtrless}} \lambda$$

3. Local Spectrum Sensing

3.1. Probability of Detection and Probability of False-Alarm

As presented in [5], there are several key parameters used to evaluate detection performance of local spectrum sensing, such as: probability of detection P_d, probability of false-alarm P_f, and probability of missed detection P_m. Probabilities of detection and false-alarm are defined as follows [5]

$$P_d = P\{Y > \lambda | H_1\} = Q_m(\sqrt{2m\gamma}, \sqrt{\lambda}) \tag{1}$$

$$P_f = P\{Y > \lambda | H_0\} = \frac{\Gamma(m, \lambda/2)}{\Gamma(m)} \triangleq G_m(\lambda) \tag{2}$$

where $\Gamma(a, b) = \int_b^\infty t^{a-1} e^{-t} dt$ is the incomplete gamma function [11] and $Q_m(.,.)$ is the generalized Marcum Q-function [12] as defined by,

$$Q_m(a, b) = \int_b^\infty \frac{x^m}{a^{m-1}} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax) dx$$

where $I_{m-1}(.)$ is the $(m-1)$-th order modified Bessel function of the first kind.

The relation between P_d and P_f is given by [5]:

$$P_d = Q_m\left(\sqrt{2m\gamma}, \sqrt{G_m^{-1}(P_f)}\right) \tag{3}$$

P_f is independent of γ and remains static since under H_0, there is no primary signal's presence. However, due to fading and shadowing, h is varying and P_d becomes conditional probability depending on the instantaneous SNR γ. In this case, the average probability of detection may be derived by averaging (3) over fading statistics,

$$P_{d,fading} = \int_\gamma Q_m\left(\sqrt{2mx}, \sqrt{G_m^{-1}(P_f)}\right) f_\gamma(x) dx \tag{4}$$

where $f_\gamma(x)$ is the probability density function (pdf) of SNR under fading.

Performance of energy detector for different values of average SNR and m may be characterized through complementary receiver operating characteristics (ROC) curves (plot of P_m vs. P_f).

3.2. Local Spectrum Sensing over Suzuki Channel

Suzuki distribution is the combination of Rayleigh and Lognormal distribution. The Suzuki distributed random variable is defined by the product of Rayleigh random variable and Lognormally distributed random variable [13]. The probability for the envelop r, of the Suzuki fading is

$$f_{R-L}(r) = \int_0^\infty \frac{r}{w^2} \exp\left(-\frac{r^2}{2w^2}\right) \times \frac{1}{\sqrt{2\pi}\sigma w} \exp\left(-\frac{(\ln w - \mu)^2}{2\sigma^2}\right) dw \tag{5}$$

where μ and σ are the parameters of Lognormal shadowing. The pdfs for the envelopes of Suzuki fading for $\mu = 0$ and various value of σ are illustrated in Figure 3. From the figure we see that, as σ decreases, the Suzuki process is more identical to the Rayleigh process.

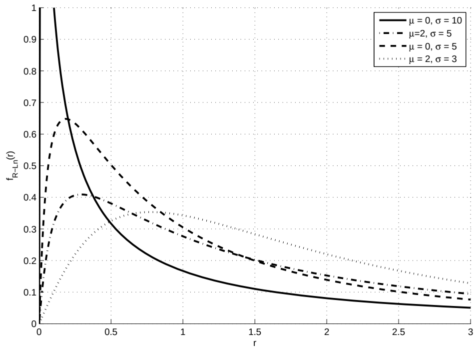

Fig. 3: The pdf of the envelope of Suzuki channel.

The pdf of Suzuki fading in term of power p, can be derived by equating the local average power of the Rayleigh faded signal to the instantaneous power of the arriving lognormal signal [14]. That means there is a complete transfer of power of the arriving lognormal signal to the local multipath channel and there is no significant loss of power in the local multipath channel, i.e. the power gain, $E[|h_R|^2] = 1$. Then, the distribution of the power gain p, of the composite fading channel is modeled as the pdf of the product of Rayleigh channels power gain and Lognormal channel's power gain,

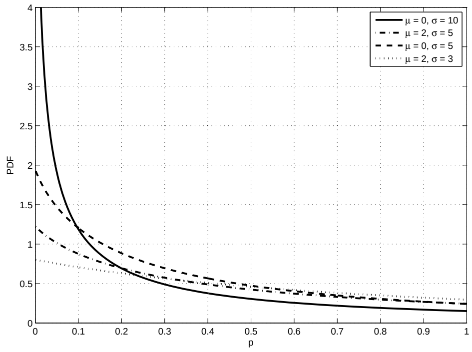

Fig. 4: The pdf of the power gain of Suzuki channel.

$$p = |h_{R-Ln}|^2 = |h_R|^2 |h_{Ln}|^2 \tag{6}$$

Using the Jacobian transformation technique, we can obtain the pdf of the power gain of the composite fading channel as (7),

$$f_{R-Ln}(p) = \int_0^\infty \frac{1}{x^2} \exp\left(\frac{p}{x}\right) \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) dx \tag{7}$$

where μ and σ are the parameters of the lognormal fading. Figure 4 illustrates the pdfs of the power of the Suzuki channels for different values of μ and σ in dB unit.

The probability of detection of Suzuki fading can be obtained by substituting f_{R-Ln} from (7) into (4),

$$P_{d,\,Suzuki} = \int_0^\infty \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) \times \left[\int_0^\infty \frac{1}{x} \exp\left(-\frac{p}{x}\right) Q_m\left(\sqrt{2mp}, \sqrt{G_m^{-1}(P_f)}\right) dp\right] dx \tag{8}$$

The expression inside the square bracket pair in (8) is the probability of detection of CR under Rayleigh fading channel (for $\bar{\gamma} = x$) which is defined as

$$P_{d,Ray} = \int_\gamma Q_m\left(\sqrt{2mp}, \sqrt{G_m^{-1}(P_f)}\right) \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right) d\gamma \tag{9}$$

where $f_\gamma(x)$ is the pdf of SNR, γ under Rayleigh fading channel. Thus, (8) can be rewritten to as

$$P_{d,Suzuki} = \int_0^\infty P_{d,Ray}(\overline{\gamma} = x) \times \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) dx \tag{10}$$

Equation (10) has the form of Gauss-Hermite integration so it can be approximated as [15],

$$P_{d,Suzuki} = \frac{1}{\sqrt{\pi}} \sum_{i=1}^{N_p} w_i P_{d,Ray}(\bar{\gamma} = e^{(\sqrt{2}\sigma a_i + \mu)}) \tag{11}$$

where a_i and w_i are the abscissas and weight factors of the Gauss-Hermite integration, and N_p is the number of samples. a_i and w_i for different values of N_p are available in [18, table (25.10)]. The bigger value of N_p, the more accurate approximation we have. The high accuracy is attained when $N_p > 6$ [16, 17].

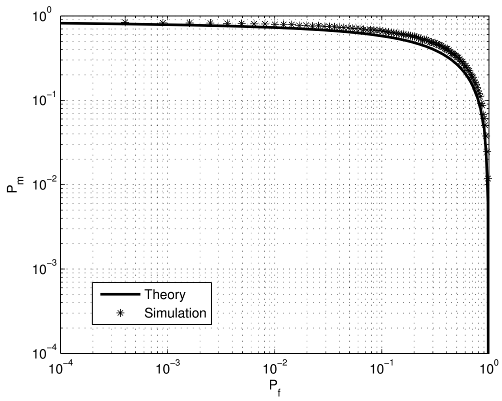

Fig. 5: The complementary ROCs under Suzuki fading.

Figure 5 illustrates the complementary ROCs under Suzuki channel for $\mu = 3dB, \sigma = 10dB$ (equivalent to $\overline{\gamma} = 14.5129dB$).

4. Cooperative Spectrum Sensing over Suzuki fading

4.1. Hard-Decision Combining

Consider a hard-decision combining in which each CR performs local spectrum sensing and sends its individual sensing information ($u_i = 0, 1$) to an FC. If $u_i = 1$, the hypothesis H_1 will be chosen, otherwise, hypothesis H_0 is chosen. The FC then collect the incoming information to come to the decision that the PU's signal is existing or not. For simplicity, we assume that:

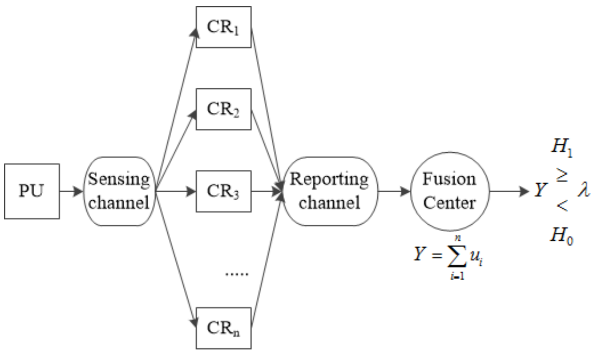

Fig. 6: The process of cooperative spectrum sensing.

- The sensing channel is affected by Suzuki fading and the sub-channels between PU and CRs are mutually independent.
- The reporting channels are ideal, that means information from CRs to PU is not lost or changed.
- The FC applies the hard-decision combining (i.e. k-out-of-n) rule.

When $k = 1, k = n$, and $k = [n/2]$, the k-out-of-n rule is also called OR rule, AND rule, and $MAJORITY$ rule, respectively. Assume that all CRs have the same value of SNR and equal probabilities of detection P_d and false-alarm P_f. Hence, the total probability of detection Q_d and the total probability of false-alarm Q_f when N CRs join the cooperative spectrum sensing [5] are:

$$Q_d = \sum_{i=k}^{n} C_n^i P_d^i (1 - P_d)^{n-i} \tag{12}$$

$$Q_f = \sum_{i=k}^{n} C_n^i P_f^i (1 - P_f)^{n-i} \tag{13}$$

where P_d and P_f were defined in (1) and (2), respectively. The total probability of missed detection is

$$Q_m = 1 - Q_d \tag{14}$$

The investigation of changes in the detection performance in cooperative spectrum sensing compared to local sensing is illustrated in Figure 7. In this case, we assume that there are 5 CRs collaborating with each other to detect the PU's signal. As we can see from the figure, the detection performance in cooperative sensing is improved significantly compared to the one in local sensing.

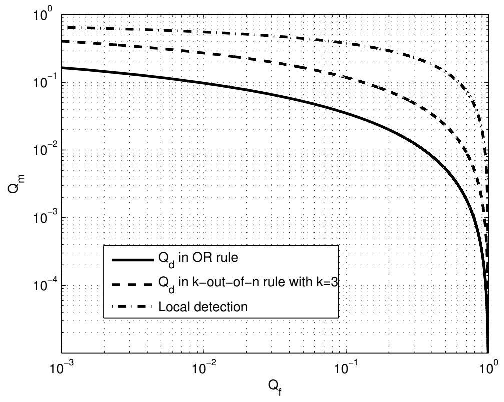

Fig. 7: The complementary ROCs under Suzuki using k-out-of-n rule with $\mu_Z = 2dB$, $\sigma_Z = 5dB$, and $n = 5$.

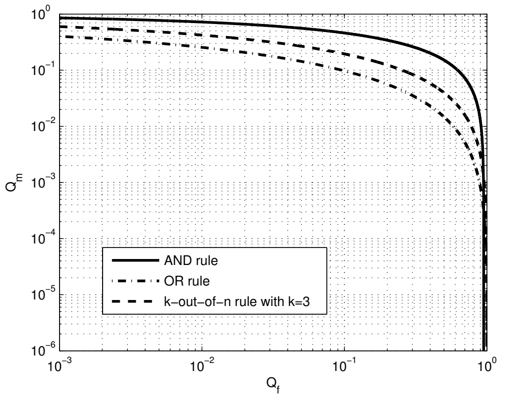

Fig. 8: The complementary ROCs under Suzuki using k-out-of-n rule ($\mu_Z = 0dB$, $\sigma_Z = 3dB$, and $n = 5$.) with various values of k.

Figure 8 illustrates the change in detection performance when we change the value of k in the k-out-of-n rule ($n = 5$ and $k = 1, 3, 5$). As can be seen from the figure, the performance degrades when k increases however, the reliability of decision (i.e., probability of detection) is better. The trade-off between the detection performance and the reliability has attracted interests of many researchers. However, we will not discuss it in this paper. Both Figures 7 and 8 show that among the k-out-of-n rules, employing *OR* rule always gives us the best detection performance. For OR rule, the FC decides H_1 when there is at least one CR user detects primary user signal, otherwise, it needs more than one. This leads to detection performance of OR rules better than other rules. Now we investigate the change of detection performance when we change the value of n ($n = 5, 7, 9$) but fix $k = 1$. As Figure 9 illustrates, when the number of CRs participating in cooperative spectrum sensing increases, the detection performance is improved considerably. However, as mentioned in section Introduction, the very large number of CRs participating in the cooperative sensing process may affect the band allocation for CRs as well as cause the overhead to the network. Therefore, harmonization between detection performance and overhead or sharing resources in the network is very necessary. This will be discussed in more details in the rest of this paper.

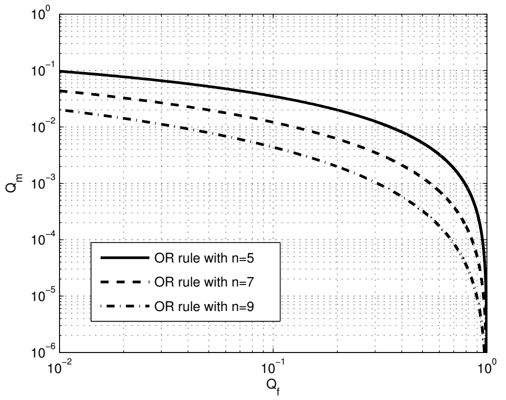

Fig. 9: The complementary ROCs under Suzuki using k-out-of-n rule ($\mu_Z = 0dB$, $\sigma_Z = 3dB$, and $k = 1$) with various values of n.

4.2. Selection of Appropriate Number of CRs in Cooperative Spectrum Sensing

In this section, we will propose a formula to find out a suitable number of cooperative CRs to avoid overhead to the network but still guarantee the detection performance with assumption that FC uses OR rule to make a global decision. Equation (12) can be rewritten as follows:

$$Q_d = 1 - (1 - P_d)^n \tag{15}$$

We observe that as $n \mapsto \infty$: $Q_d \mapsto 1$. Let ε be a very small number so that when n increases to a certain value, the condition $1 - Q_d < \varepsilon$ is always satisfied. Thus,

$$Q_d = \sum_{i=1}^{n} C_n^i P_d^i (1 - P_d)^{n-i} = 1 - (1 - P_d)^n \geq 1 - \varepsilon \tag{16}$$

or

$$\varepsilon \geq 1 - Q_d = Q_m = (1 - P_d)^n \tag{17}$$

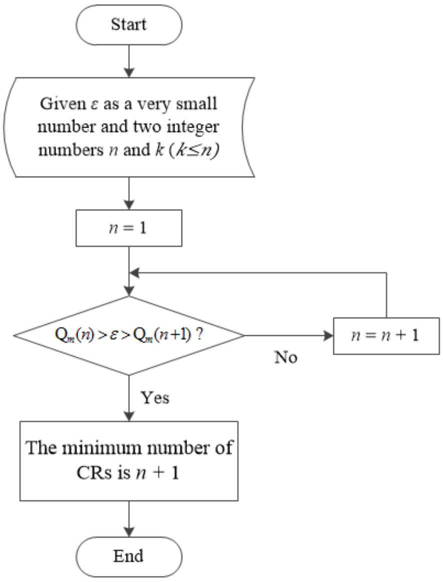

Fig. 10: The flow chart for choosing appropriate number of CRs in cooperative spectrum sensing.

Generally, the formula of the number of CRs joining cooperative spectrum sensing is

$$n = \min\{\arg\{\varepsilon \geq Q_m\}\} \tag{18}$$

For a given value of ε, we can apply the following algorithm to compute the minimum value of n satisfying (18)

- For given values of P_f, n and k, we can compute the corresponding Q_d.
- Set 1 as the initial value of n.
- Increase n until (18) is satisfied, that is

$$Q_m(n) > \varepsilon > Q_m(n+1) \tag{19}$$

- The minimum number of CRs is $n + 1$.

The algorithm can be illustrated by a flow chart as given in Figure 10 above. Figure 11 shows the detection performance under composite fading vs. number of CRs taking part in the collaborative spectrum sensing under Suzuki channel. Obviously, as n becomes large, Q_f is approximated to 1. For $\varepsilon = 10^{-3}$, we can find the number of CRs as the results shown in the figure. With these results, not only the detection performance is guaranteed at a required threshold value but also the network can avoid much overhead.

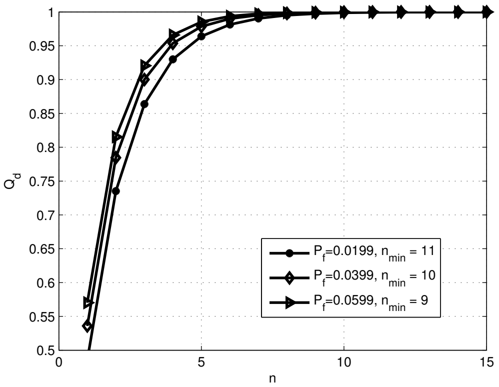

Fig. 11: The detection performance by number of cooperative CRs under Suzuki channel using *OR* rule with $\varepsilon = 10^{-3}$.

For comparison purposes, we also provide the detection performance vs. number of CRs under Rayleigh and Lognormal channels in Figure 12. Note that, the average power gains of three kinds of fadings are the same, i.e, $\overline{p}_{Suzuki} = \overline{p}_{Rayleigh} = \overline{p}_{Lognormal}$, in which Suzuki and Lognormal variables have the same Gaussian parameters with $\mu = 2$ dB and $\sigma = 5$dB. As can be seen from this figure, Rayleigh and lognormal channels require fewer CRs taking part in the cooperative spectrum sensing process than Suzuki channel. This is because Suzuki channel is the composition of both Rayleigh and

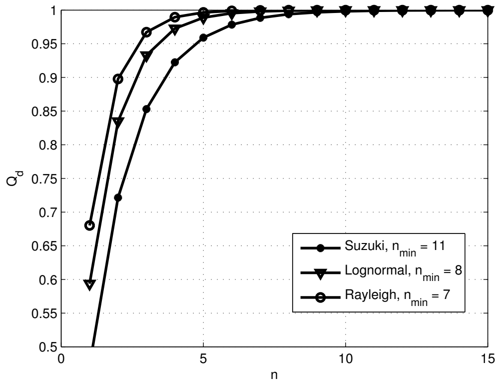

Fig. 12: The detection performance by number of cooperative CRs under Rayleigh, Lognormal, and Suzuki channels using *OR* rule with $\varepsilon = 10^{-3}$.

lognormal channels and therefore, it is more complicated than its component channels. In details, the considered Suzuki variables consist of two components: lognormal variable which has the same average power gain and Rayleigh one with average power gain equal to 1 (i.e. 0 dB) as mentioned in Section 3.2. Rayleigh component is the cause of the degradation in detection performance of cooperative spectrum sensing under Suzuki fading when compared to that under lognormal fading which have the same average power gain. The results above are compatible with the characteristics and the complexity of these three channels.

5. Conclusion

Cooperative spectrum sensing is one of the very effective ways to enhance the detection performance of CRs in wireless channels. In this paper, we have investigated the performance of cooperative spectrum sensing over Suzuki fading channels based on Hard-Decision Combining rule and compared it to the local spectrum sensing. Numerical results show that cooperative technique provides better performance than what the local on does. Besides, in collaborative spectrum sensing, employing *OR* rule gives us higher probability of detection compared to *AND* rule and non-cooperative signal detection at different SNR values. Furthermore, for $\varepsilon = 10^{-3}$ and $P_f \geq 0.0199$, a minimum of 11 collaborated CRs relatively in cognitive radio system can achieve the optimal value of probability of detection.

With the space constraint of the paper, we only consider the performance of cooperative spectrum sensing with the assumption of free-loss physical links between cooperating CRs and FC which are so-called reporting channels. The effect of Suzuki fading on these channels for investigating cooperative detection performance will be taken into account in our further work.

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